SCET sum rules for $B \to P$ and $B \to V$ transition form factors

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ABSTRACT: We investigate sum rules for heavy-to-light transition form factors at large recoil derived from correlation functions with interpolating currents for light pseudoscalar or vector fields in soft-collinear effective theory (SCET). We consider both, factorizable and non-factorizable contributions at leading power in the Λ/m_b expansion and to first order in the strong coupling constant α_s, neglecting contributions from 3-particle distribution amplitudes in the B-meson. We pay particular attention to various sources of parametric and systematic uncertainties. We also discuss certain form factor ratios where part of the hadronic uncertainties related to the B-meson distribution amplitude and to logarithmically enhanced α_s corrections cancel.

KEYWORDS: Heavy Quark Physics, QCD, B-Physics.

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Contents

1. Introduction

The phenomenological analysis of semi-leptonic and non-leptonic B-meson decays into light mesons needs non-perturbative hadronic input. The theoretically simplest objects are the heavy-to-light transition form factors. At large recoil energy ($E \sim m_B/2$) and to leading power in the Λ/m_B expansion, they fulfill a factorization theorem [1] which can be proven using effective-field-theory methods [2, 3] (see also [4]):

$$\langle M(E)|\bar{\psi}\,\Gamma_i\,b|B(v)\rangle = C_i^{\mathrm{I}}(\mu, 2E)\,\xi_M(2E, \mu) + C_i^{\mathrm{II}}(\mu, 2E)\,\frac{\alpha_s C_F}{4\pi}\,\Delta F_M(2E, \mu) + \dots, \quad (1.1)$$

where $M = P, V_\parallel, V_\perp$. The factorization theorem contains a so-called "soft" (non-factorizable) function ξ_M and a factorizable piece ΔF_M which is determined by a convolution of a spectator-scattering kernel T and process-independent light-cone distribution amplitudes (LCDAs) $\phi_B^+(\omega)$ and $\phi_M(u)$ for heavy and light mesons, respectively,

$$\Delta F_M(2E, \mu) = T(u, 2E, \omega, \mu) \otimes \phi_B^+(\omega, \mu) \otimes \phi_M(u, \mu). \qquad (1.2)$$

Finally, the perturbative coefficient functions $C_i^{\mathrm{I,II}}$ contain the radiative corrections from short-distance quantum fluctuations at the scale $\mu \sim m_b$.

The modelling of the soft form-factor terms ξ_M requires non-perturbative methods. Because of the *qualitatively different* dynamics parametrized by the functions ξ_M and ΔF_M, it is desirable to follow a non-perturbative approach where the two contributions can clearly be separated from the very beginning of the calculation. In previous work [5, 6] we have shown that ξ_M and ΔF_M can be determined independently via sum rules derived from correlation functions in soft-collinear effective theory (SCET [7, 8]), where the light mesons are replaced by suitably chosen interpolating currents. Both, the correlation function for ξ_M and for ΔF_M involve the light-cone distribution amplitudes of the B-meson. Radiative corrections to the correlation function can be systematically calculated in perturbation theory, separating the dynamics at the intermediate ("hard-collinear") scale of order $\sqrt{m_b \Lambda_{\mathrm{QCD}}}$ from the soft hadronic binding effects in the B-meson. It is to be stressed that the correlation functions unambiguously factorize into perturbative short-distance functions and process-independent LCDAs (the factorization to order α_s accuracy and neglecting 3-particle LCDAs will be verified explicitly below). In contrast, the sum rules derived from these correlators introduce additional sensitivity to non-perturbative parameters, which not necessarily needs to be process-independent and thus reflects some irreducible theoretical uncertainty.

In our previous article we have concentrated on the $B \to \pi$ form factor. In this work we extend and generalize the discussion to also include transitions to light vector mesons, $V = \rho, K^*, \ldots$ We find it particularly useful to consider form factor ratios, where the leading dependence on the hadronic input parameters related to the B-meson drops out to some extent.

The paper is organized as follows: In section 2 we derive the SCET sum rules for the $B \to V_\parallel, V_\perp$ non factorizable form factors at $\mathcal{O}(\alpha_s)$. The sum rules for the corresponding factorizable form factors are obtained in section 3. Numerical analyses are performed in section 4, where also the case of $B \to P$ form factors is reconsidered. The last section is devoted to conclusions. Some useful formulas are collected in the appendix.

2. SCET sum rule for non-factorizable form factor

In the following, we will use light-cone variables defined in terms of two light-like vectors $n_+^2 = n_-^2 = 0$ which are normalized as $n_+ n_- = 2$, such that any vector is decomposed as

$$a^\mu = (n_+ a) \frac{n_-^\mu}{2} + (n_- a) \frac{n_+^\mu}{2} + a_\perp^\mu. \qquad (2.1)$$

We consider a reference frame where the B-meson velocity vector satisfies $v_\perp = 0$ and $n_+ v = n_- v = 1$, and the final-state momentum p' is purely longitudinal, $p'_\perp = 0$. In this frame the two independent kinematic variables appearing in the SCET correlation functions (see below) are taken as

$$(n_+ p') \simeq 2E' = \mathcal{O}(m_b), \qquad 0 > (n_- p') = \mathcal{O}(\Lambda). \tag{2.2}$$

The dispersive analysis will be performed with respect to $(n_- p')$ for fixed values of $(n_+ p')$.

In SCET the non-factorizable form factors for transitions between a B meson and a light vector meson are defined in terms of matrix elements of the two current operators [2]

$$J_0^\parallel = \bar{\xi}_{\text{hc}} W_{\text{hc}} \left(-\gamma_5 \right) Y_s^\dagger h_v, \tag{2.3}$$

$$J_0^{\nu_\perp} = \bar{\xi}_{\text{hc}} W_{\text{hc}} \left(\gamma^{\nu_\perp} \right) Y_s^\dagger h_v, \tag{2.4}$$

where ξ_{hc} is the "good" light-cone component of the light-quark spinor with $\not{n}_- \xi_{\text{hc}} = 0$, and h_v is the usual HQET field. The hard-collinear and soft Wilson lines, W_{hc} and $Y_s^\dagger$, appear to render the form-factor definitions manifestly gauge invariant in SCET. The normalization conventions for the corresponding two form factors $\xi_\parallel$ and $\xi_\perp$ are as in [1]

$$\langle V(p', \varepsilon)|J_0^\parallel|\bar{B}(v)\rangle = \frac{(n_+ \varepsilon^*)}{2} (n_+ p') \, \xi_\parallel(n_+ p'), \tag{2.5}$$

$$\langle V(p', \varepsilon)|J_0^{\nu_\perp}|\bar{B}(v)\rangle = \frac{i}{2} (n_+ p') \, \xi_\perp(n_+ p') \, \epsilon^{\nu_\perp \kappa_\perp \sigma\tau} \varepsilon^*_{\kappa_\perp} n_{-\tau} n_{+\sigma} \tag{2.6}$$

where $\epsilon^{0123} = +1$.

The SCET sum rules are derived from a dispersive analysis of the correlation functions

$$\Pi_\parallel(n_- p') = i \int d^4 x \, e^{ip'x} \, \langle 0|T[J_V^\parallel(x) J_0^\parallel(0)]|B(v)\rangle, \tag{2.7}$$

$$\frac{1}{2} \epsilon^{\mu_\perp \nu_\perp \sigma\tau} n_{+\sigma} \, n_{-\tau} \, \Pi_\perp(n_- p') = i \int d^4 x \, e^{ip'x} \, \langle 0|T[J_V^{\mu_\perp}(x) J_0^{\nu_\perp}(0)]|B(v)\rangle, \tag{2.8}$$

where the longitudinal and transverse polarization-state of the light vector meson is replaced by the interpolating current

$$J_V^\parallel(x) = -i \, \bar{\xi}_{\text{hc}}(x) \not{n}_+ \xi_{\text{hc}}(x) - i \left(\bar{\xi}_{\text{hc}} W_{\text{hc}}(x) \not{n}_+ Y_s^\dagger q_s(x) + \text{h.c.} \right), \tag{2.9}$$

and

$$i J_V^{\mu_\perp}(x) = \bar{\xi}_{\text{hc}}(x) i \not{n}_+ \gamma^{\mu_\perp} \xi_{\text{hc}}(x) + \left(\bar{\xi}_{\text{hc}} W_{\text{hc}}(x) i \not{n}_+ \gamma^{\mu_\perp} Y_s^\dagger q_s(x) + \text{h.c.} \right), \tag{2.10}$$

respectively. Here we denoted soft quark fields in SCET as q_s. Notice that soft-collinear interactions require a multi-pole expansion of soft fields [8] which is always understood implicitly. The matrix element of the interpolating vector-meson currents are given as

$$\langle 0|J_V^\parallel|V(p', \varepsilon)\rangle = m_V (n_+ \varepsilon) f_V^\parallel, \tag{2.11}$$

$$\langle 0|i J_V^{\mu_\perp}|V(p', \varepsilon)\rangle = (n_+ p') \varepsilon^{\mu_\perp} f_V^\perp(\mu). \tag{2.12}$$

The dispersion relation for the above correlation functions, after Borel transform with respect to the variable n_-p', reads

$$\hat{B}\left[\Pi_{\|,\perp}\right](\omega_M) = \int\limits_0^\infty d\omega' \, \frac{1}{\omega_M} \, e^{-\omega'/\omega_M} \, \frac{1}{\pi} \, \text{Im}\left[\Pi_{\|,\perp}(\omega')\right] \,, \tag{2.13}$$

where $\omega_M \equiv M^2/(n_+p')$ is the Borel parameter. The hadronic representation of the same correlation function reads

$$\Pi_{\|,\perp}^{\text{HAD}}(n_-p') = \Pi_{\|,\perp}(n_-p')\Big|_{\text{res.}} + \Pi_{\|,\perp}(n_-p')\Big|_{\text{cont.}} \,, \tag{2.14}$$

where the first term represents the contribution of the V meson, while the second takes into account the role of higher states and continuum above an effective threshold $\omega_s \equiv s_0/(n_+p')$. Assuming quark-hadron duality implies $\Pi_{\|,\perp}(n_-p') = \Pi_{\|,\perp}^{\text{HAD}}(n_-p')$. For the longitudinal part we obtain the resonance contribution

$$\Pi_\|(n_-p')\Big|_{\text{res.}} = \frac{\langle 0|J_V^\| |V(p',\varepsilon)\rangle\langle V(p',\varepsilon)|J_0^\| |B(v)\rangle}{m_V^2 - p'^2} = \frac{n_+p'}{2m_V} \, \frac{(n_+p')^2 \, \xi_\|(n_+p') \, f_V^\|}{m_V^2 - p'^2} \,. \tag{2.15}$$

Apart from an overall factor $(n_+p')/2m_V = E_V/m_V$ this has the same form as the expression for $B \to \pi$ if one replaces $(f_\pi, \xi_\pi) \to (f_V^\|, \xi_V^\|)$. However, now we should not neglect the vector-meson mass in the hadronic side of the sum rule. As a consequence, the Borel transform of the resonance contribution reads

$$\hat{B}\left[\Pi_\|\right](\omega_M)\Big|_{\text{res.}} = \frac{n_+p'}{2m_V} \exp\left[-\frac{m_V^2}{(n_+p')\,\omega_M}\right] \frac{(n_+p')\,\xi_\|(n_+p')\,f_V^\|}{\omega_M} \,. \tag{2.16}$$

Modelling the continuum by the perturbative result and subtracting it on both sides of (2.14), we obtain the sum rule

$$\frac{n_+p'}{2m_V} \, \xi_\|(n_+p') = \frac{1}{f_V^\|(n_+p')} \exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right] \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \frac{1}{\pi}\text{Im}[\Pi_\|(\omega')] \,. \tag{2.17}$$

Analogously, the sum rule for the transverse form factor follows as

$$\xi_\perp(n_+p') = \frac{1}{f_V^\perp(n_+p')} \exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right] \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \frac{1}{\pi}\text{Im}[\Pi_\perp(\omega')] \,. \tag{2.18}$$

2.1 Tree-level result

At leading power, the tree-level result for the correlation function involves one hard-collinear quark propagator, which reads

$$S_F^{\text{hc}} = \frac{i}{n_-p' - \omega + i\eta} \, \frac{\not{n}_-}{2} \,, \tag{2.19}$$

where $\omega = n_-k$, and k^μ is the momentum of the soft light quark that will end up as the spectator quark in the B meson. The remaining matrix element defines light-cone

distribution amplitudes for the B meson in HQET [9, 1] (for the definition, see appendix B). This leads to

$$\Pi_\parallel(n_-p') = \Pi_\perp(n_-p') = f_B m_B \int\limits_0^\infty d\omega \, \frac{\phi_B^-(\omega)}{\omega - n_-p' - i\eta} \tag{2.20}$$

which is identical to the analogous result calculated for the $B \to \pi$ case [5]. Inserting the imaginary part into the sum rules (2.17), (2.18), the tree-level result for the soft form factors follows as

$$\hat{\xi}_\parallel(n_+p') \equiv \frac{n_+p'}{2m_V} \xi_\parallel(n_+p')$$

$$= \frac{f_B m_B}{f_V^\parallel (n_+p')} \exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right] \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \phi_B^-(\omega') \quad \text{(tree level)}, \tag{2.21}$$

and

$$\xi_\perp(n_+p') = \frac{f_B m_B}{f_V^\perp (n_+p')} \exp\left[\frac{m_V^2}{(n_+p')\,\omega_M}\right] \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \phi_B^-(\omega') \quad \text{(tree level)}. \tag{2.22}$$

Notice that the suppression of the longitudinal form factor $\xi_\parallel$ by $m_V/(n_+p')$ compared to $\xi_\perp$ is expected on general grounds [10, 1].

2.2 Radiative corrections from hard-collinear loops

In SCET short-distance radiative corrections to the correlation functions (2.7), (2.8) are represented by hard-collinear loops, as shown in figure 1 for the leading order in α_s (1-loop contributions from three-particle distribution amplitudes in the B-meson are neglected in this work). The diagrams denoted by (a1-a4) and (b1-b2) form gauge-invariant subsets, such that the result for either of the correlation functions can be written as

$$\Pi_{\parallel,\perp}(\omega', \mu) = f_B(\mu) m_B \int\limits_0^\infty \frac{d\omega}{\omega - \omega' - i\eta} \, \phi_B^-(\omega, \mu) \left\{ 1 + \frac{\alpha_s C_F}{4\pi} \left((\text{a1-a4}) + (\text{b1-b2}) \right) \right\}_{\parallel,\perp}. \tag{2.23}$$

The corrections from diagrams (a1-a4) are identical for pseudoscalar, longitudinal and transverse vector mesons and read [5]

$$(\text{a1-a4}) = \frac{4}{\epsilon^2} + \frac{3 + 4L(\mu)}{\epsilon} + L(\mu)\,(3 + 2L(\mu)) + 7 - \frac{\pi^2}{3}, \tag{2.24}$$

where we have used dimensional regularization in $D = 4 - 2\epsilon$, and defined the abbreviation

$$L(\mu) = \ln\left[-\frac{\mu^2}{(n_+p')(\omega' - \omega + i\eta)} \right]. \tag{2.25}$$
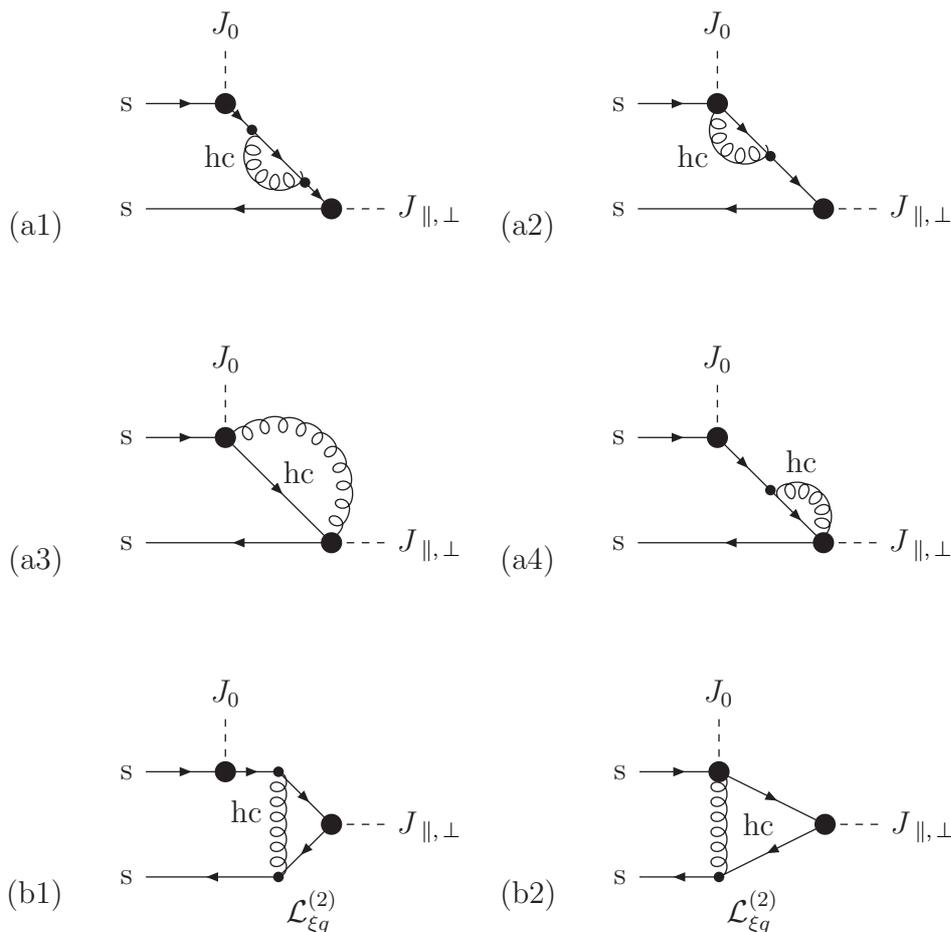
Figure 1: Diagrams contributing to the sum rule for $\xi_{\parallel,\perp}$ to order α_s with hard-collinear loops and no external soft gluons. Diagrams (a2-a4) and (b2) vanish in light-cone gauge $n_+ A_{\mathrm{hc}} = 0$. Diagram (a3) vanishes both in light-cone and in Feynman gauge.

Notice that the diagrams (a1-a4) also determine the perturbative corrections to the jet function in inclusive $b \to u$ decays [11, 12] (see also appendix).

For the diagrams (b1-b2) we have to consider soft-collinear vertices from the sub-leading SCET$_\mathrm{I}$ Lagrangian $\mathcal{L}_{\xi q}^{(2)}$, see [8]. Alternatively, one can single out the hard-collinear integration region from the corresponding QCD diagrams and use the momentum-space projector for the B-meson distribution amplitude, see (B.7) in the appendix. In either case we obtain

$$(\mathrm{b1} - \mathrm{b2})_{\parallel} = -\frac{2}{\epsilon^2} - \frac{2L_1 + 2L(\mu) + 3}{\epsilon}$$
$$- L_1^2 - \frac{\omega + 2\omega'}{\omega} L_1 - L^2(\mu) - (2L_1 + 3)L(\mu) + \frac{\pi^2}{6} - 8 \,, \qquad (2.26)$$

where we have defined

$$L_1 = \ln\left[1 - \frac{\omega}{\omega' + i\eta}\right] \,. \qquad (2.27)$$

Again, the expression for $(b_1 - b_2)_\parallel$ coincides with the $B \to \pi$ case.[1] For the transverse polarization we obtain

$$(\text{b1-b2})_\perp = (\text{b1-b2})_\parallel - \frac{1}{\epsilon} - L(\mu) - L_1 \,. \tag{2.28}$$

The extra contribution with $1/\epsilon$ is related to the anomalous dimension of the tensor current (2.10), and the corresponding extra μ dependence cancels that of $f_V^\perp(\mu)$,

$$\frac{\partial f_V^\perp(\mu)}{\partial \ln \mu^2} = \frac{\alpha_s C_F}{4\pi} f_V^\perp(\mu) + \dots \tag{2.29}$$

2.2.1 Cancellation of factorization-scale dependence

The physical form factors defined by (1.1) have to be independent of the factorization scale μ, which have been introduced to separate the momentum regions contributing to the $B \to M$ transition. In our previous paper [5] we have already verified that the double-logarithmic dependence on μ, related to the universal cusp anomalous dimension, drops out when combining the scale-dependence of the hard matching coefficients between QCD and SCET [7],

$$\frac{d}{d \ln \mu} C_i(\mu) = -\frac{\alpha_s C_F}{4\pi} \left(\Gamma_{\text{cusp}}^{(1)} \ln \frac{\mu}{m_b} + 5 \right) C_i(\mu) + \dots \tag{2.30}$$

with $\Gamma_{\text{cusp}}^{(1)} = 4$, and the explicit scale dependence of the renormalized SCET correlation function in (2.23),

$$\frac{d}{d \ln \mu} \Pi_\parallel(\omega', \mu) = -\frac{\alpha_s C_F}{4\pi} f_B(\mu) m_B \int_0^\infty \frac{d\omega}{\omega - \omega'} \int_0^\infty d\tilde{\omega} \, \gamma_-^{(1)}(\omega, \tilde{\omega}, \mu) \, \phi_B^-(\tilde{\omega}, \mu) \tag{2.31}$$

$$+ \frac{\alpha_s C_F}{4\pi} f_B(\mu) m_B \int_0^\infty \frac{d\omega}{\omega - \omega'} \left(\Gamma_{\text{cusp}}^{(1)} L(\mu) - \Gamma_{\text{cusp}}^{(1)} L_1 + 3 \right) \phi_B^-(\omega, \mu) + \dots$$

Here we have only quoted the result for $\Pi_\parallel$, since $\Pi_\perp$ simply differs by a single log related to the anomalous dimension of the decay constant $f_\perp(\mu)$ (see above). Neglecting possible 3-particle contributions, the scale dependence of the B-meson LCDA in HQET is described by the anomalous dimension γ_-,

$$\frac{d}{d \ln \mu} \phi_B^-(\omega; \mu) = -\frac{\alpha_s C_F}{4\pi} \int_0^\infty d\tilde{\omega} \, \gamma_-^{(1)}(\omega, \tilde{\omega}; \mu) \, \phi_B^-(\tilde{\omega}; \mu) + \dots \tag{2.32}$$

which gives rise to the first line in (2.31). The second line arises from the explicit scale-dependence induced by the NLO hard-collinear α_s corrections to the correlator, where we also took into account the anomalous dimension of the B-meson decay constant in HQET, $df_B/d \ln \mu = 3 \frac{\alpha_s C_F}{4\pi} + \dots$

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[1] We spotted a calculational error in our original result for the $B \to \pi$ correlator in [5], which affects the single-logarithmic terms. The corrected formulas and numerics will be discussed below.

While the anomalous dimension for the LCDA $\phi_B^+(\omega, \mu)$ has been known for some time [13], the anomalous dimension for the LCDA $\phi_B^-(\omega, \mu)$ has been calculated only recently [14] with the result

$$\gamma_-^{(1)}(\omega, \tilde{\omega}; \mu) = \left(\Gamma_{\text{cusp}}^{(1)} \ln \frac{\mu}{\omega} - 2 \right) \delta(\omega - \tilde{\omega}) - \Gamma_{\text{cusp}}^{(1)} \frac{\theta(\tilde{\omega} - \omega)}{\tilde{\omega}}$$
$$- \Gamma_{\text{cusp}}^{(1)} \omega \left[\frac{\theta(\tilde{\omega} - \omega)}{\tilde{\omega}(\tilde{\omega} - \omega)} \right]_+ - \Gamma_{\text{cusp}}^{(1)} \omega \left[\frac{\theta(\omega - \tilde{\omega})}{\omega(\omega - \tilde{\omega})} \right]_+ . \tag{2.33}$$

This enables us to also verify the perturbative cancellation of the single logarithmic terms. For that purpose, we have to insert the convolution (2.32) into the tree-level term in (2.23). Using integration-by-parts, we find

$$- \int_0^\infty \frac{d\omega}{\omega - \omega'} \int_0^\infty d\tilde{\omega} \, \gamma_-^{(1)}(\omega, \tilde{\omega}, \mu) \, \phi_B^-(\tilde{\omega}, \mu)$$
$$= \int_0^\infty \frac{d\omega}{\omega - \omega'} \left(-\Gamma_{\text{cusp}}^{(1)} \ln \frac{\mu}{\omega - \omega'} + \Gamma_{\text{cusp}}^{(1)} L_1 + 2 \right) \phi_B^-(\omega, \mu) . \tag{2.34}$$

Inserting (2.34) into (2.31) and combining with (2.30), we find indeed complete cancellation of all μ-dependent terms, which proofs the factorization of the SCET correlator to order α_s accuracy (in the absence of 3-particle LCDAs).

2.2.2 Contribution to the sum rule

In [5] we have calculated the imaginary part of the correlation function using the approximation $\phi_B^-(\omega) \simeq \phi_B^-(0)$ which is valid for $\omega_{M,s} \ll \Lambda$. Using the relations provided in the appendix, it is possible to derive the exact expressions which, inserted in eq. (2.13), give the final result for the Borel-transformed and continuum-subtracted correlator:

$$\hat{B}[\Pi_{\|,\perp}](\omega_M) - \text{cont.} = \frac{f_B m_B}{\omega_M} \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \left\{ - \int_{\omega'}^\infty d\omega \, f_{\|,\perp}(\omega, \omega', \mu) \frac{d\phi_B^-(\omega, \mu)}{d\omega} \right.$$
$$\left. + \int_0^{\omega'} d\omega \left[\frac{g_{\|,\perp}(\omega, \omega', \mu)}{\omega - \omega'} \right]_+ \phi_B^-(\omega, \mu) \right\} \tag{2.35}$$

where we introduced the functions $f_{\|,\perp} = 1 + \mathcal{O}(\alpha_s)$ and $g_{\|,\perp} = \mathcal{O}(\alpha_s)$:

$$f_\|(\omega, \omega', \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left(L_0^2 + (1 + 2L_0) \ln \frac{\omega'}{\omega} - (3 + 2L_0) \ln \left[1 - \frac{\omega'}{\omega} \right] - 1 + \frac{\pi^2}{6} \right) \tag{2.36}$$

$$f_\perp(\omega, \omega', \mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left(L_0^2 - L_0 + (2 + 2L_0) \ln \frac{\omega'}{\omega} - (4 + 2L_0) \ln \left[1 - \frac{\omega'}{\omega} \right] - 1 + \frac{\pi^2}{6} \right) \tag{2.37}$$

and

$$g_\|(\omega, \omega', \mu) = g_\perp(\omega, \omega', \mu) + \frac{\alpha_s C_F}{4\pi} = \frac{\alpha_s C_F}{4\pi} (2L_0 - 4L_1) , \tag{2.38}$$

where

$$L_0 = \ln\left[\frac{\mu^2}{(n_+p')\omega'}\right] .$$

(2.39)

The final sum rules at $\mathcal{O}(\alpha_s)$ are obtained by replacing $\phi_B^-(\omega')$ in (2.21), (2.22) by the curly brackets in (2.35),

$$\phi_B^-(\omega') \to \phi_{\parallel,\perp}^{\text{eff}}(\omega', n_+p', \mu) \equiv \left\{ -\int_{\omega'}^{\infty} d\omega\, f_{\parallel,\perp}(\omega, \omega', \mu)\, \frac{d\phi_B^-(\omega, \mu)}{d\omega} \right.$$
$$\left. + \int_0^{\omega'} d\omega \left[\frac{g_{\parallel,\perp}(\omega, \omega', \mu)}{\omega - \omega'}\right]_+ \phi_B^-(\omega, \mu) \right\}$$

(2.40)

Notice that the integration variable in the dispersion integral is restricted to values $\omega' \leq \omega_s = s_0/(n_+p')$, and therefore the natural choice for the scale μ in the function L_0 is soft, $\mu_{\text{soft}}^2 \sim s_0 = \mathcal{O}(1\,\text{GeV}^2)$ (i.e. independent of the heavy-quark mass). On the other hand, the logarithm $\ln[\omega'/\omega]$ in the function $f(\omega, \omega', \mu)$ is evaluated for values of $\omega \geq \omega'$, and therefore the natural scale for ω is set by the B-meson LCDA, $\omega \sim \omega_0 \simeq 0.5$ GeV. The identified large logarithms, arising in the formal limit $\mu^2 \sim s_0 \ll \omega_0(n_+p')$, stem from the spectator diagrams (b1,b2) in figure 1 and have the same origin as the endpoint singularities appearing in the QCD factorization approach.[2] The appearance of different scales in ϕ_{eff} thus indicates that the resummation of large logarithms in the heavy-quark limit *for the sum rule* is not complete.

A formal solution of this problem within the sum-rule approach would require to consider two-loop corrections to the correlator and to better understand the evolution equations of the B-meson LCDA $\phi_B^-(\omega)$ in the presence of 3-particle LCDAs, which goes beyond the scope of this work. Usually, one takes a more pragmatic point of view and ignores the formal logarithmic enhancement of the perturbative coefficients, since numerically the effect does not appear to be very large. However, one should be aware of the fact that the presence of these non-factorizable logarithms prevents us from performing the very heavy-quark limit in the final sum rule for the soft form factor, see also section 4.2.2 below.

3. SCET sum rule for QCD-factorizable contributions

In our previous paper [5], we have shown that at leading-order in α_s our sum-rule approach reproduces the result for the *factorizable* form-factor contribution from hard-collinear spectator scattering as derived in [2, 1] (spectator-scattering corrections to heavy-to-light form factors at NLO in QCD factorization are also known by now [16 – 18]). We extend the results derived in [5] for $B \to \pi$ transition to the case of B decays to a light vector meson.

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[2]A similar situation arises for heavy-to-light form factors with non-relativistic bound states, as discussed in [15]. Here, the finite (constituent) quark masses provide an intrinsic infrared regulator, such that the form factors are well-defined in fixed-order perturbation theory. Still, one encounters large logarithms which cannot be attributed to the evolution of mesonic light-cone wave function or hard interaction kernels.
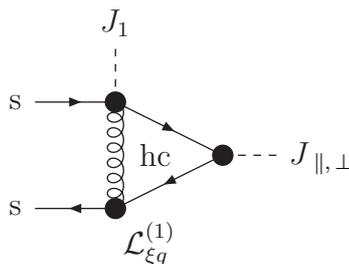
Figure 2: Factorizable contribution to heavy-to-light form factors in the SCET sum rule approach.

In case of the pion the corrections to the symmetry relations are derived by an independent sum rule starting with the correlation function

$$\Pi^1_\pi(p') = i \int d^4x \, e^{ip'x} \, \langle 0|T[J_\pi(x)J_1^\pi(0)]|B(p_B)\rangle \,, \tag{3.1}$$

where

$$J_1^\pi \equiv \bar{\xi}_{\text{hc}} \, g \, \slashed{A}_{\text{hc}}^\perp \, h_v \,. \tag{3.2}$$

The analogous function for the ρ with longitudinal polarization is

$$\Pi^1_\parallel(p') = i \int d^4x \, e^{ip'x} \, \langle 0|T[J_\rho^\parallel(x)J_1^\parallel(0)]|B(v)\rangle \,, \tag{3.3}$$

with the factorizable current[3]

$$J_1^\parallel \equiv \bar{\xi}_{\text{hc}} \, g \, \slashed{A}_{\text{hc}}^\perp \, \gamma_5 h_v \,. \tag{3.4}$$

In the case of ρ with transverse polarization there are two possible currents having a nonvanishing matrix element between the B meson and a transverse ρ state, which are

$$J_{1,1}^{\nu_\perp} \equiv \bar{\xi}_{\text{hc}} \, g \, A_{\text{hc}}^{\nu_\perp} h_v \,, \quad J_{1,5}^{\nu_\perp} \equiv \bar{\xi}_{\text{hc}} \, g \, A_{\text{hc}}^{\nu_\perp} \gamma_5 h_v \,. \tag{3.5}$$

As a consequence, we consider two different correlation functions:

$$\Pi_\perp^{1,1\mu_\perp\nu_\perp} = i \int d^4x \, e^{ip'x} \, \langle 0|T[J_\rho^{\mu_\perp}(x)J_{1,1}^{\nu_\perp}(0)]|B(v)\rangle = (-1)\frac{1}{2} \, \epsilon_\perp^{\nu\mu} \, \Pi_\perp^{1,1}(p') \tag{3.6}$$

with $\epsilon_\perp^{\nu\mu} \equiv \epsilon^{\nu_\perp\mu_\perp\sigma\tau} \, n_{-\tau} v_\sigma$, or alternatively

$$\Pi_\perp^{1,5\mu_\perp\nu_\perp} = i \int d^4x \, e^{ip'x} \, \langle 0|T[J_\rho^{\mu_\perp}(x)J_{1,5}^{\nu_\perp}(0)]|B(v)\rangle = (-i)\frac{1}{2} \, g_\perp^{\mu\nu} \, \Pi_\perp^{1,5}(p') \,. \tag{3.7}$$

The leading contributions to those correlation functions are given by the diagram in figure 2 which involves the insertion of one interaction vertex from the order-λ soft-collinear Lagrangian

$$\mathcal{L}_{\xi q}^{(1)} = \bar{\xi}_{\text{hc}} \, g_s \slashed{A}_{\text{hc},\perp} \, q_s + \text{h.c.} \,.$$

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[3]Note that this current could be in principle further reduced to $J_1^\parallel \equiv i\epsilon_\perp^{\mu\sigma} \, \bar{\xi}_{\text{hc}} \, g \, A_{\text{hc}\,\mu}^\perp \, \gamma_\sigma^\perp h_v$.

As in the pion case, the resulting hard-collinear loop integrals are UV- and IR-finite and all the scalar functions turn out to be equal:

$$\Pi^1(p') \equiv \Pi^1_\pi(p') = \Pi^1_\parallel(p') = \Pi^{1,1}_\perp(p') = \Pi^{1,5}_\perp(p') \,. \tag{3.8}$$

The calculation yields

$$\Pi^1(p') = -\frac{\alpha_s C_F}{4\pi}(n_+ p') \int_0^\infty d\omega \, \frac{f_B m_B \, \phi^+_B(\omega)}{\omega} \, \ln\left[1 - \frac{\omega}{n_- p' + i\eta}\right] \,, \tag{3.9}$$

the imaginary part of which reads as

$$\frac{1}{\pi} \operatorname{Im}[\Pi^1(\omega')] = -\frac{\alpha_s C_F}{4\pi}(n_+ p') \int_0^\infty d\omega \, \frac{f_B m_B \, \phi^+_B(\omega)}{\omega} \, \theta[\omega - \omega'] \,. \tag{3.10}$$

Inserting this into the dispersion relation analogous to the one of the tree-level sum rule

$$\Pi^1(p') = \frac{1}{\pi} \int_0^\infty d\omega' \, \frac{\operatorname{Im}[\Pi_1(\omega')]}{\omega' - n_- p' - i\eta} \,, \tag{3.11}$$

and performing the Borel transformation and subtracting the continuum contribution, we obtain

$$\hat{B}\left[\Pi^1\right](\omega_M)\Big|_{\text{res.}} = -\frac{\alpha_s C_F}{4\pi}(n_+ p') \int_0^\infty d\omega \, \frac{f_B m_B \, \phi^+_B(\omega)}{\omega}$$
$$\left(1 - e^{-\omega_s/\omega_M} - \theta(\omega_s - \omega)\left(e^{-\omega/\omega_M} - e^{-\omega_s/\omega_M}\right)\right) \,. \tag{3.12}$$

Let us first reconsider the pion case. The hadronic expression for the contribution of the lowest lying resonance (the pion) reads in this case

$$\Pi^1_\pi(p')\Big|_{\text{res.}} = \frac{\langle 0|J_\pi|\pi(p')\rangle\langle\pi(p')|J^\pi_1|B(p_B)\rangle}{m_\pi^2 - p'^2} = \frac{(n_+ p') \, f_\pi \, \langle\pi(p')|J^\pi_1|B(p_B)\rangle}{m_\pi^2 - p'^2} \,. \tag{3.13}$$

Thus, its Borel transform is given by

$$\hat{B}\left[\Pi^1_\pi\right](\omega_M)\Big|_{\text{res.}} = \frac{f_\pi \langle\pi|J^\pi_1|B\rangle}{\omega_M} \, e^{-m_\pi^2/(n_+ p' \omega_M)} \,. \tag{3.14}$$

Equating the two expressions leads to the sum rule for the factorizable contribution in the pion case which reduces to the result derived in [5] for $m_\pi = 0$:

$$\langle\pi|J^\pi_1|B\rangle = -\frac{\alpha_s C_F}{4\pi} \, \frac{(n_+ p') f_B m_B \omega_M}{f_\pi} \, e^{m_\pi^2/(n_+ p' \omega_M)} \tag{3.15}$$
$$\times \int_0^\infty d\omega \, \frac{\phi^+_B(\omega)}{\omega} \left(1 - e^{-\omega_s/\omega_M} - \theta(\omega_s - \omega)\left(e^{-\omega/\omega_M} - e^{-\omega_s/\omega_M}\right)\right) \,.$$

In previous work [1], the factorizable contribution was expressed in terms of the quantities $\Delta F_\pi, \Delta F_\parallel$, and $\Delta F_\perp$. As was explicitly shown in the appendix of [5], one can identify the matrix element corresponding to the factorizable contribution as follows

$$\langle\pi|J^\pi_1|B\rangle = \frac{\alpha_s C_F}{4\pi} \, \Delta F_\pi \frac{(-m_B^2)}{2} \,. \tag{3.16}$$

Along the same lines, one derives the identifications of the quantities $\Delta F_\parallel$ and $\Delta F_\perp$ with the matrix elements of the factorizable currents defined above:

$$\langle \pi | J_1^\parallel | B \rangle = \frac{\alpha_s C_F}{4\pi} \Delta F_\parallel \frac{(-m_B^2)}{4} n_+ \epsilon^* \frac{m_\rho}{E} \tag{3.17}$$

$$\langle \pi | J_{1,1}^{\nu\perp} | B \rangle = \frac{\alpha_s C_F}{4\pi} \Delta F_\perp \frac{(-im_B^2)}{4} \epsilon_{\kappa\perp}^* \epsilon_\perp^{\nu\kappa} \tag{3.18}$$

$$\langle \pi | J_{1,5}^{\nu\perp} | B \rangle = \frac{\alpha_s C_F}{4\pi} \Delta F_\perp \frac{(m_B^2)}{4} \epsilon^{*\nu\perp} . \tag{3.19}$$

Then the hadronic side of the sum rule can be directly expressed in terms of the ΔF_X: Using again $|n_+\epsilon^*|^2 = (n_+p')^2/(m_\rho)^2$ we get

$$\hat{B}\left[\Pi_\parallel^1\right](\omega_M)\Big|_{\text{res.}} = \hat{B}\left[\frac{f_\rho^\parallel n_+p'}{m_\rho^2 - p'^2} \frac{(-m_B^2)}{2} \frac{\alpha_s C_F}{4\pi} \Delta F_\parallel\right] \tag{3.20}$$

$$= \frac{f_\rho^\parallel}{\omega_M} e^{-m_\rho^2/(n_+p'\omega_M)} \frac{(-m_B^2)}{2} \frac{\alpha_s C_F}{4\pi} \Delta F_\parallel , \tag{3.21}$$

and using $\epsilon^{*\mu\perp}\epsilon^{*\nu\perp} = -g_\perp^{\mu\nu}$, we get for the scalar functions in the transverse case, $\hat{B}\left[\Pi^{1,1}\right](\omega_M)\Big|_{\text{res.}}$ and $\hat{B}\left[\Pi^{1,5}\right](\omega_M)\Big|_{\text{res.}}$, the same result as if we replace $f_\rho^\parallel$ and $\Delta F_\parallel$ by $f_\rho^\perp$ and $\Delta F_\perp$ respectively.

Equating the hadronic and SCET expressions in the various cases we end up with the following sum rule for the quantities ΔF_X where $X = \pi, \rho_\parallel, \rho_\perp$:

$$\Delta F_X(\mu, n_+p') = \frac{2f_B \omega_M (n_+p')}{m_B f_X} e^{m_X^2/(n_+p'\omega_M)} \tag{3.22}$$

$$\times \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu) \left(1 - e^{-\omega_s/\omega_M} \theta(\omega - \omega_s) - e^{-\omega/\omega_M} \theta(\omega_s - \omega)\right)$$

(with ω_M and ω_s depending on the considered meson X).

This result can now be compared with QCD factorization [2, 1]. If we insert the leading-order sum rule for the decay constants

$$4\pi^2 f_X^2 \simeq M^2 e^{m_X^2/M^2} \left(1 - e^{-s_0/M^2}\right) , \tag{3.23}$$

where $M^2 \equiv \omega_M (n_+p')$ and $s_0 \equiv \omega_s (n_+p')$, we can write the above result as

$$\Delta F_X(\mu, n_+p') = \frac{8\pi^2 f_B f_X}{m_B} \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu) \times \tag{3.24}$$

$$\times \frac{1 - e^{-\frac{s_0}{M^2}} \theta\left(\omega - \frac{s_0}{n_+p'}\right) - e^{-\omega n_+p'/M^2} \theta\left(\frac{s_0}{n_+p'} - \omega\right)}{1 - e^{-\frac{s_0}{M^2}}} .$$

Notice that, in contrast to the case for the non-factorizable form factor ξ_M, the limit $s_0 \ll \omega\, n_+p'$ now does exist. This implies that the soft dynamics related to the B-meson constituents decouples from the dynamics related to the spectrum of the interpolating current. Therefore, it is indeed justified to identify the sum rule parameters in the otherwise

independent sum rules for ΔF_X and the corresponding decay constant f_X. Not surprisingly, in this limit the above formula (3.24) reduces to the prediction from QCD factorization, with — at this order — the asymptotic form for the light meson LCDAs $\phi_X(u)$,

$$\Delta F_X(\mu)\Big|_{\text{QCDF,asympt.}} = \frac{8\pi^2 f_B f_X}{m_B} \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega, \mu)\,. \tag{3.25}$$

4. Numerical predictions for form factors

4.1 Hadronic input parameters

The result for the soft form factors from the SCET sum rules depends on the following hadronic input parameters:

- The B-meson distribution amplitude $f_B\,\phi_B^-(\omega, \mu)$: In the following, we will relate $1/\omega_0 = \phi_B^-(0)$ to $\lambda_B = \Big[\int_0^\infty d\omega\, \frac{\phi_+(\omega)}{\omega}\Big]^{-1}$ via the Wandzura-Wilczek approximation [1] at the low scale $\mu = 1\,\text{GeV}$. We take the values which have been obtained from a recent moment analysis in [19]

  $$\omega_0(1\ \text{GeV}) = (0.48 \pm 0.05)\ \text{GeV}\,.$$

  (The value is consistent with the sum-rule result from [20].) For the shape of $\phi_B^-(\omega)$, we adopt the simple parametrization

  $$\phi_B^-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0}\,, \tag{4.1}$$

  see, however, also the discussion in section 4.5.

- The B-meson decay constant is taken as $f_B(m_b) = (180 \pm 30)\,\text{MeV}$ which corresponds to $f_B(1\ \text{GeV}) \simeq (150 \pm 30)\,\text{MeV}$.

- The threshold parameter $\omega_s = s_0/(n_+ p')$: The next vector-meson resonance above the ρ meson is the $\rho(1450)$, and therefore we may expect $\omega_s \sim 0.4\,\text{GeV}$ at $(n_+ p') = m_B$. We will take $\omega_s = \{0.35, 0.4, 0.45\}\,\text{GeV}$ as our default range for longitudinal ρ-mesons.[4]

  In case of the correlation function $\Pi_\perp$ for transversely polarized ρ mesons, also the axial-vector resonances may contribute, starting with $b_1(1235)$. As a consequence, the threshold parameter in our sum rule for $\xi_\perp$ is expected to be significantly smaller than in the longitudinal case (see, for instance, the discussion in [21]). Therefore, we consider the lower threshold values, $\omega_s = \{0.20, 0.25, 0.30\}\,\text{GeV}$ for transverse ρ-mesons.

  For completeness, we will also reconsider the soft $B \to \pi$ form factor, for which we will take $\omega_s = \{0.15, 0.20, 0.25\}\,\text{GeV}$.

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[4]In the following, all values assigned to the sum-rule parameters ω_s and ω_M refer to the maximal recoil point $(n_+ p') = m_b$. However, when $(n_+ p')$ or $m_Q \neq m_b$ are varied, these parameters are re-scaled accordingly.

- The Borel parameter ω_M: Reasonable values of ω_M should be estimated from the sum rule itself. As it turns out, the prediction for the soft form factors approximately grows logarithmically with ω_M. Requiring that the sum rule is sufficiently stable against variations of ω_M thus determines a *lower* acceptable value for ω_M. In practice, we consider the normalized logarithmic derivative

$$D = \frac{\omega_M}{\xi_{\|,\perp,\pi}} \frac{\partial \xi_{\|,\perp,\pi}}{\partial \omega_M} \qquad (4.2)$$

and require $|D| < 25\%$ (however, we generally do not consider values of ω_M below $m_\rho^2/m_b \simeq 0.1\,\text{GeV}$). As a further constraint, we impose that the continuum contribution ($\omega' > \omega_s$) is not too large, and require that

$$R = \frac{\int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \phi^{\text{eff}}(\omega')}{\int\limits_0^{\infty} d\omega' \, e^{-\omega'/\omega_M} \, \phi^{\text{eff}}(\omega')} \qquad (4.3)$$

is larger than 50%.

- For the meson decay constants[5] we take $f_\rho^\| = 205\,\text{MeV}$, and $f_\rho^\perp(\mu = 1 \text{ GeV}) = 160\,\text{MeV}$ [21].

4.2 Soft form factor for longitudinal vector mesons

Figure 3(a) shows the dependence of the soft form factor $\hat{\xi}_\|$ at $n_+p' = m_B$ and $\mu = 1.0\,\text{GeV}$ as a function of the Borel parameter for three different values of the threshold parameters. Our constraints $R_\| > 50\%$ and $D < 25\%$ are fulfilled for a rather large range of Borel parameter values, $0.1 \text{ GeV} < \omega_M < 3.35 \text{ GeV}$, and we take the geometric mean $\omega_M = 0.6 \text{ GeV}$ as our central value. Taking into account the variation of the various input parameters, we obtain the estimate

$$\hat{\xi}_\|(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.33^{+0.02}_{-0.02}|_{\omega_s}{}^{+0.03}_{-0.06}|_{\omega_M}{}^{+0.03}_{-0.02}|_{\omega_0} \pm 0.05|_{f_B}. \qquad (4.4)$$

Adding errors in quadrature amounts to $\hat{\xi}_\|(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.33^{+0.07}_{-0.09}$, i.e. in total a $\sim 25\%$ uncertainty with the largest contribution coming from the Borel parameter and the decay constant. For comparison, the tree level result for the same input parameters would give $\hat{\xi}_\| = 0.38$, illustrating the numerical significance of the α_s corrections. Within the rather large uncertainties, our value is compatible with the values obtained from traditional sum rules [22, 23]. (For instance, the phenomenological analysis in [24] (see table 2 therein) infers $\hat{\xi}_\|(m_B, 1.5 \text{ GeV}) = 0.37 \pm 0.06$.)

In principle, one can decrease the sensitivity on the sum rule parameters by dividing the sum rule for $\xi_\|$ by the leading-order sum rule for the decay constant f_ρ,

$$4\pi^2 f_\rho^2 \, e^{-m_\rho^2/M^2} \simeq \int_0^{s_0} ds \, e^{-s/M^2} ,$$

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[5]In the following, we do not quote the uncertainty w.r.t. the decay constant $f_\rho^\perp$ which amounts to about 5-10% and would propagate linearly into the form factor sum rule.
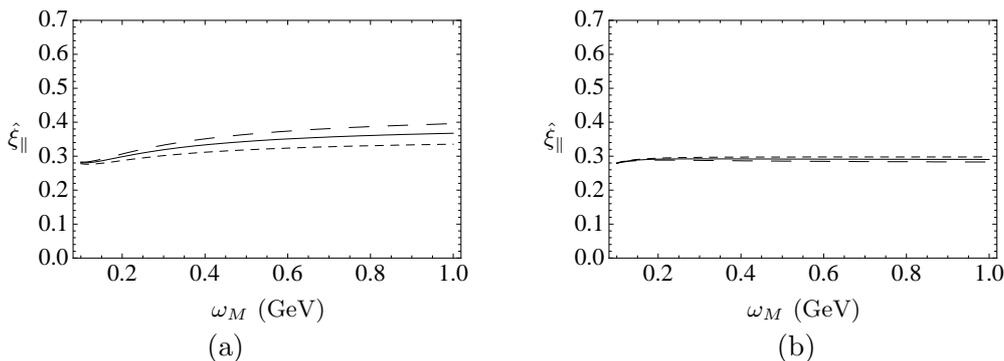
Figure 3: (a) The soft form factor $\hat{\xi}_\parallel$ at $n_+p' = m_B$ and $\mu = 1.0\,\text{GeV}$ as a function of the Borel parameter for three different values of the threshold parameters (solid line: $\omega_s = 0.4\,\text{GeV}$, short-dashed line: $\omega_s = 0.45\,\text{GeV}$, long-dashed line: $\omega_s = 0.35\,\text{GeV}$). (b) The same quantity using the modified sum rule (4.5).

where, for simplicity, we have not included α_s corrections and non-perturbative corrections from quark and gluon condenstates, which are suppressed by powers of $1/M^2$. If we *assume* that the parameters $\omega_M = M^2/(n_+p')$ and $\omega_s = s_0/(n_+p')$ in the two sum rules can be identified, we obtain a modified sum rule

$$\hat{\xi}_\parallel(n_+p', \mu) \simeq \frac{4\pi^2 f_\rho f_B m_B \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, \phi_\parallel^{\text{eff}}(\omega', n_+p', \mu)}{(n_+p')^2 \int\limits_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M}} \, . \tag{4.5}$$

Numerically, the modified sum rule would result in the estimate

$$\hat{\xi}_\parallel(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.29 \pm 0.01|_{\omega_s}{}_{-0.01}^{+0.00}|_{\omega_M} \pm 0.02|_{\omega_0} \pm 0.05|_{f_B} \pm (?)_{\text{syst.}} \, .$$

The result is plotted in figure 3(b). It is to be stressed, however, that the very fact that the soft and collinear dynamics in $\xi_\parallel$ do not factorize in QCD also implies that the sum rule parameters for the $B \to \rho$ form factor and the ρ-meson decay constant are not trivially correlated, and the above numerical value is only shown for illustration. The reduction of the ω_M dependence in (4.5) is probably compensated by an increased systematic error (indicated by the question mark) which is hard to estimate reliably. Nevertheless, we will find the modified sum rules useful in the context of form factor ratios (see section 4.6.1), where one could argue that the systematic error from correlating parameters in different sum rules and neglecting higher-order corrections drops out to some extent.

4.2.1 Energy dependence

We may also study the energy dependence of the soft form factor, which is of phenomenological importance if one wants to interpolate between lattice-theory results at intermediate values of q^2 and sum-rule results for the large-recoil limit at $q^2 = 0$ (for a comprehensive discussion, see e.g. [25]). The normalized result for the energy dependence is shown in
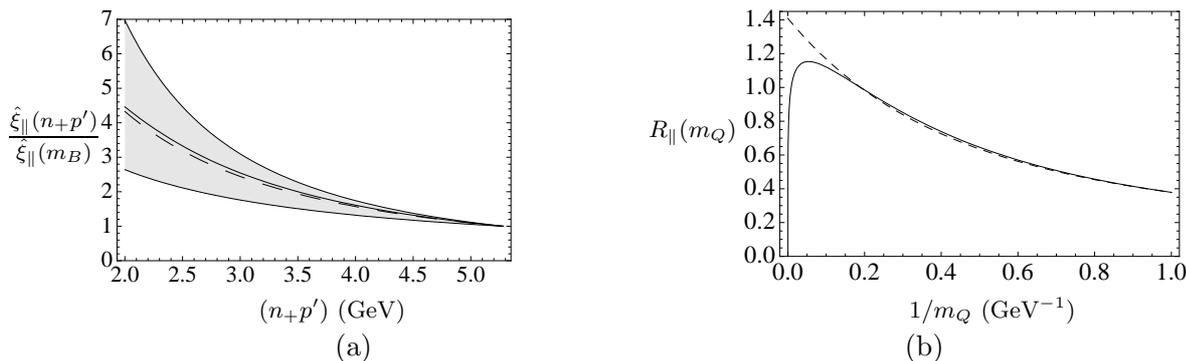
Figure 4: (a) Energy dependence of the soft form factor $\hat{\xi}_\parallel(n_+p')/\hat{\xi}_\parallel(m_B)$. The grey band illustrates the range between a pure $1/(n_+p')^2$ and a pure $1/(n_+p')$ behaviour. (b) The m_Q dependence of the soft form factor $R_\parallel(m_Q)$, see (4.6), as a function of $1/m_Q$. The solid line denotes the NLO result, the dashed line the LO result.

figure 4(a), where we have also shown a pure $1/(n_+p')$ and $1/(n_+p')^2$ behaviour for comparison. Here we considered the sum rule parameters to scale with the recoil energy according to $M^2 = \omega_M (n_+p') = $ const. and $s_0 = \omega_s (n_+p') = $ const. We observe that the resulting energy dependence is just between a $1/(n_+p')$ and a $1/(n_+p')^2$ fall off. We also note that the difference between the energy dependence of the tree-level and the NLO result is tiny. We have also studied the stability criteria of the sum rule as a function of (n_+p') and found that the ω_M dependence is not changed very much, while the continuum pollution is even decreasing with smaller values of (n_+p'). We therefore consider the predictions for the energy dependence of the soft form factor $\hat{\xi}_\parallel$ to be rather solid.

4.2.2 m_Q dependence

The scaling of the soft form factor with the heavy-quark mass is another interesting issue. From the theoretical side one is interested in the competition between the soft Feynman mechanism for intermediate values of m_Q and the Sudakov supression in the asymptotic limit $m_Q \to \infty$. Phenomenologically, the heavy-quark mass scaling could be exploited to get a rough estimate for D-meson decays at large recoil by extrapolation from the B-meson case. In figure 4(b) we study the m_Q dependence of the normalized soft form factor at maximal recoil $(n_+p') = m_Q$,

$$R_\parallel(m_Q) \equiv \frac{m_Q}{m_B} \frac{f_B(\mu)}{f_Q(\mu)} \frac{\hat{\xi}_\parallel(E_{\max})|_{m_Q}}{\hat{\xi}_\parallel(E_{\max})|_{m_B}}, \tag{4.6}$$

where the factorization scale is kept fixed at $\mu = 1.0 \,\text{GeV}$, and the pre-factors take into account the "trivial" m_Q dependence. Again, the sum rule parameters are scaled such that $M^2 = \omega_M \, m_Q = $ const. and $s_0 = \omega_s \, m_Q = $ const.

For values $m_Q < m_b$, the situation is analogous to the energy dependence $(n_+p' < m_B)$ discussed above, i.e. an extrapolation of the sum rule to smaller heavy quark masses seems to be justified. However, the $1/m_Q$ power corrections become, of course, even more
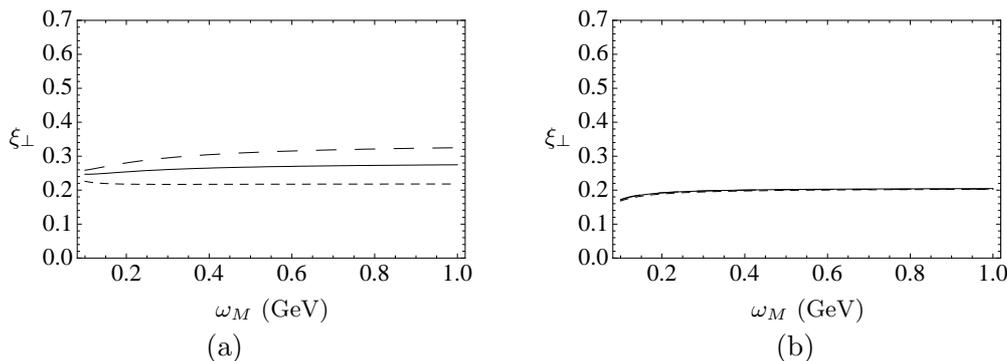
Figure 5: (a) The soft form factor $\xi_\perp$ at $n_+p' = m_B$ and $\mu = 1.0\,\text{GeV}$ as a function of the Borel parameter for three different values of the threshold parameters (solid line: $\omega_s = 0.25\,\text{GeV}$, short-dashed line: $\omega_s = 0.2\,\text{GeV}$, long-dashed line: $\omega_s = 0.3\,\text{GeV}$). (b) The same quantity using the modified sum rule (4.8).

important in that case (see also the discussion in [26]). On the other hand, for larger values of m_Q, the sum rule predictions becomes more and more unstable. This manifests itself in: (i) a huge difference between the tree-level and the NLO result in the limit $m_Q \to \infty$, (ii) an increased sensitivity to the sum rule parameters, (iii) an increased continuum pollution. In particular, the very heavy quark limit $m_Q \to \infty$ for $\hat{\xi}_\parallel$ does not exist anymore beyond the tree-level approximation, which can be traced back to the non-factorizable large logarithms discussed in section 2.2.2. However, one has to keep in mind that only the product of the soft form factor $\hat{\xi}_\parallel(n_+p', \mu)$ and the matching coefficient $C_i^{\text{I}}(\mu, n_+p')$ between the QCD and the SCET current contributes to the physical form factor in (1.1), and $C_i^{\text{I}}(\mu, m_Q)$ vanishes in the limit $m_Q \to \infty$ (for fixed values of μ) due to the exponentiation of Sudakov logs [7].

4.3 Soft form factor for transverse vector mesons

The analysis of the form factors for B-decays into transversely polarized vector mesons is similar as for longitudinal ones. The main difference is the smaller value for the default threshold parameter ω_s, due to the contribution of the $b_1(1235)$ to the correlator $\Pi_\perp$. The numerical result for the form factor $\xi_\perp$ is shown in figure 5(a). Considering the stability of the sum-rule result for $\xi_\perp$ and the smallness of the continuum contribution, we find that also the Borel parameter ω_M has to be chosen in a range smaller than in the longitudinal case, $0.1 < \omega_M < 0.8\,\text{GeV}$, with the central value taken as $\omega_M = 0.3\,\text{GeV}$. Our result for the soft transverse $B \to \rho$ form factor follows as

$$\xi_\perp(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.26^{+0.03}_{-0.04}\big|_{\omega_s}{}^{+0.01}_{-0.01}\big|_{\omega_M} \pm 0.03\big|_{\omega_0} \pm 0.04\big|_{f_B} \qquad (4.7)$$

Adding errors in quadrature amounts to $\xi_\perp(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.26^{+0.06}_{-0.07}$, i.e. in total a $\sim 25\%$ uncertainty with similar contributions from the different input parameters. The tree-level result for the same input parameters gives $\xi_\perp = 0.39$. (For comparison, in [24] the value $\xi_\perp(\mu = 1.5 \text{ GeV}) = 0.27 \pm 0.05$ has been derived from the traditional sum rule approach in [22].)
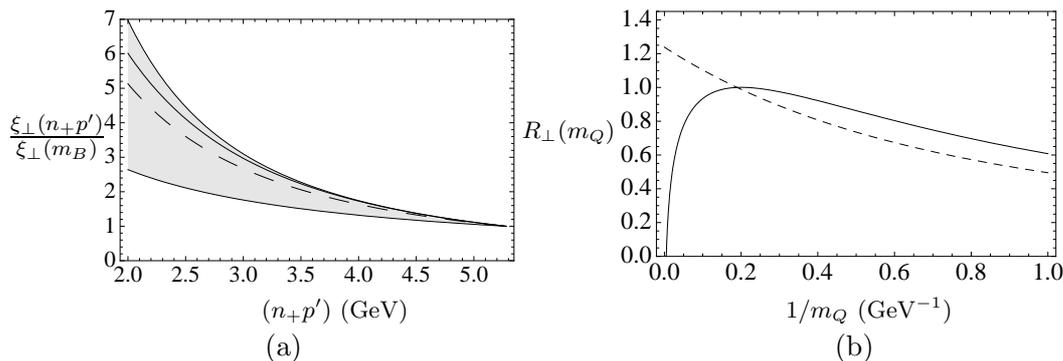
Figure 6: (a) Energy dependence of the soft form factor $\xi_\perp(n_+p')/\xi_\perp(m_B)$. The grey band illustrates the range between a pure $1/(n_+p')^2$ and a pure $1/(n_+p')$ behaviour. (b) The m_Q dependence of the soft form factor $R_\perp(m_Q)$, (defined analogously as (4.6)), as a function of $1/m_Q$. The solid line denotes the NLO result, the dashed line the LO result.

If we divide the sum rule for $\xi_\perp$ by the corresponding sum rule for $f_\rho^\perp$ [21],

$$4\pi^2\, (f_\rho^\perp)^2\, e^{-m_\rho^2/M^2} \simeq \int_0^{s_0} ds\, e^{-s/M^2} \left(1 + \frac{\alpha_s}{\pi}\left[\frac{7}{9} + \frac{2}{3}\ln\frac{s}{\mu^2}\right]\right),$$

we obtain the modified prediction

$$\xi_\perp(n_+p',\mu) = \frac{4\pi^2 f_\rho^\perp f_B m_B \int\limits_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \phi_\perp^{\rm eff}(\omega', n_+p', \mu)}{(n_+p')^2 \int\limits_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\left(1 + \frac{\alpha_s}{\pi}\left(\frac{7}{9} + \frac{2}{3}\ln\frac{\omega' n_+p'}{\mu^2}\right)\right)}, \tag{4.8}$$

where we also took into account the explicit μ dependence arising from the $\mathcal{O}(\alpha_s)$ correction to the tensor current. Again, the modified sum rule reduces the parametric uncertainties related to ω_s and ω_M, but induces an unknown systematic error due to the assumed correlation between the two sum rules,

$$\xi_\perp(n_+p' = m_B, \mu = 1.0 \text{ GeV}) = 0.20^{+0.00}_{-0.00}|_{\omega_s}{}^{+0.01}_{-0.03}|_{\omega_M} \pm 0.02|_{\omega_0} \pm 0.03|_{f_B} \pm (?)_{\rm syst}.$$

Notice that the central value obtained from the modified sum rule is significantly smaller than the one from the original sum rule (and only marginally consistent within the uncertainties). In figure 6 we show again the energy and heavy-mass dependence. Compared to the longitudinal case, the energy dependence of $\xi_\perp$ is somewhat closer to a $1/(n_+p')^2$ fall-off. Also the heavy-quark mass dependence is slightly different.

4.4 Soft form factor for pseudoscalar mesons

For completeness, we also show the results for the soft $B \to \pi$ form factor, where the sum rule takes the same form as for longitudinally polarized vector mesons (with $m_\pi^2 \approx 0$),

$$\xi_\pi(n_+p',\mu) = \frac{f_B m_B}{f_\pi (n_+p')} \int\limits_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \phi_\parallel^{\rm eff}(\omega', n_+p', \mu). \tag{4.9}$$
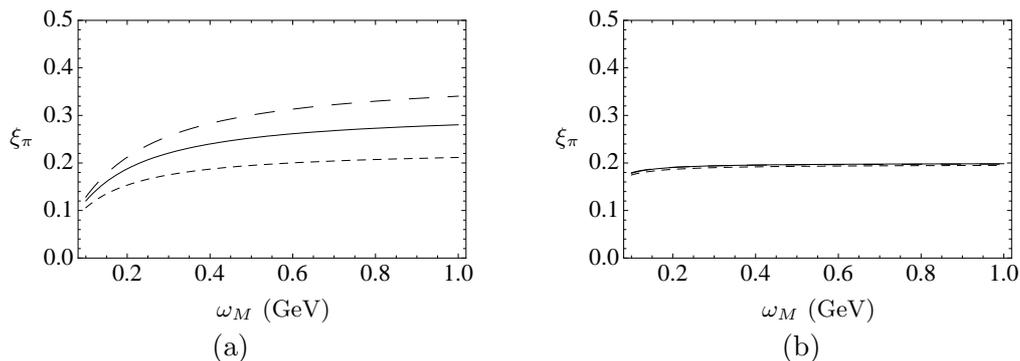
Figure 7: (a) The soft form factor ξ_π at $n_+ p' = m_B$ and $\mu = 1.0$ GeV as a function of the Borel parameter for three different values of the threshold parameters (solid line: $\omega_s = 0.2$ GeV, short-dashed line: $\omega_s = 0.25$ GeV, long-dashed line: $\omega_s = 0.15$ GeV). (b) The same quantity using the modified sum rule (4.11).

The choice for the threshold parameter s_0 is somewhat more difficult in this case, since the axial-vector current, used to interpolate the pion, is expected to receive sizeable contributions from multi-pion channels. Typical values for s_0 used in sum rules for, say, the pion decay constant are thus rather low and lie in the range $0.7 - 1.0$ GeV^2, corresponding to $\omega_s = 0.15 - 0.20$ GeV. On the other hand, one might argue that the Feynman mechanism for the soft $B \to X$ form factor is dominant for single-particle states X, while multi-particle decays $B \to \pi\pi\pi$ etc. should be suppressed. In this case, one would expect values of s_0 in the $B \to \pi$ sum rule close to the first axial-vector resonance. In the following, we choose a rather conservative range $\omega_s = \{0.15, 0.2, 0.25\}$ GeV (in accordance with our previous work [5]). Consequently, our result for ξ_π will have a rather large uncertainty from the variation of s_0.

As for the Borel parameter, we again consider the stability of ξ_π with respect to variations of ω_M and the amount of continuum pollution within the given range for ω_s. From $D < 25\%$ and $R > 50\%$ we infer 0.3 GeV $< \omega_M < 0.7$ GeV with the central value chosen as $\omega_M = 0.45$ GeV. Notice that the allowed range for ω_M is somewhat smaller than in the vector meson case, which is due to the rather small values of s_0. With even smaller values of s_0 (like e.g. the ones chosen in [27]), the criteria $D < 25\%$ and $R > 50\%$ could not be simultaneously fulfilled anymore. Moreover, for too small values of ω_s the convergence of the perturbative series for the sum rule is bad, since the α_s corrections are enhanced by large logarithms $\ln \omega_s/\omega_0$ (see the discussion above). This again could be taken as an indication that the considered lower values for s_0 are less realistic.

In any case, our choice for the variation of the sum-rule parameters implies the following estimate for ξ_π at maximal recoil,

$$\xi_\pi(n_+ p' = m_B, \mu = 1.0 \text{ GeV}) = 0.25^{+0.05}_{-0.06}|_{\omega_s}{}^{+0.02}_{-0.03}|_{\omega_M}{}^{+0.03}_{-0.02}|_{\omega_0} \pm 0.04|_{f_B}. \quad (4.10)$$

Adding errors in quadrature amounts to $\xi_\pi(n_+ p' = m_B, \mu = 1.0 \text{ GeV}) = 0.25^{+0.07}_{-0.08}$, i.e. in total a $\sim 30\%$ uncertainty with the largest contributions coming from the decay constant
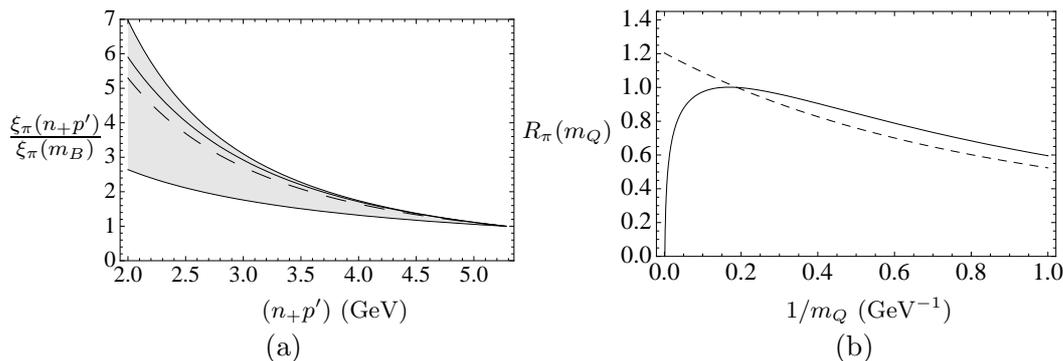
Figure 8: (a) Energy dependence of the soft form factor $\xi_\pi(n_+p')/\xi_\pi(m_B)$. The grey band illustrates the range between a pure $1/(n_+p')^2$ and a pure $1/(n_+p')$ behaviour. (b) The m_Q dependence of the soft form factor $R_\pi(m_Q)$, (defined analogously as (4.6)), as a function of $1/m_Q$. The solid line denotes the NLO result, the dashed line the LO result.

and the threshold parameter. (Again, we quote the corresponding tree-level result, $\xi_\pi = 0.32$, for comparison).

The modified sum rule is obtained by dividing the sum rule for f_π,

$$\xi_\pi(n_+p',\mu) \simeq \frac{4\pi^2 f_\pi f_B m_B \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}\, \phi_\parallel^{\mathrm{eff}}(\omega',n_+p',\mu)}{(n_+p')^2 \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M}}. \tag{4.11}$$

Numerically, we obtain

$$\xi_\pi(n_+p' = m_B, \mu = 1.0\ \mathrm{GeV}) = 0.20^{+0.00}_{-0.00}\big|_{\omega_s}{}^{+0.00}_{-0.00}\big|_{\omega_M} \pm 0.02\big|_{\omega_0} \pm 0.03\big|_{f_B} \pm (?)\big|_{\mathrm{syst.}}.$$

In figure 8 we show the energy and heavy-quark mass dependence of ξ_π which is similar to that of $\xi_\perp$.

4.5 Dependence on the shape of $\phi_B^-(\omega)$

So far, in our error treatment, we have varied the parameter $\omega_0 = 1/\phi_B^-(0)$ in a conservative range in order to estimate the uncertainties related to the B-meson LCDA. It should be stressed, however, that our result also depends on the *shape* of $\phi_B^-(\omega)$ in an essential way. To illustrate this effect, we consider two alternative parametrizations for $\phi_B^-(\omega)$

$$1: \qquad \phi_B^-(\omega) = \frac{1}{\omega_0} \exp\left[-\left(\frac{\omega}{\omega_1}\right)^2\right], \qquad\qquad \omega_1 = \frac{2\,\omega_0}{\sqrt{\pi}}; \tag{4.12}$$

$$2: \qquad \phi_B^-(\omega) = \frac{1}{\omega_0}\left(1 - \sqrt{\left(2 - \frac{\omega}{\omega_2}\right)\frac{\omega}{\omega_2}}\right)\theta(\omega_2 - \omega), \qquad \omega_2 = \frac{4\,\omega_0}{4 - \pi}. \tag{4.13}$$

Both models have the same value at $\omega = 0$ as our default model (4.1) and are normalized to unity (the radiative tail [19] of $\phi_B^-(\omega)$ is unimportant for our considerations, because
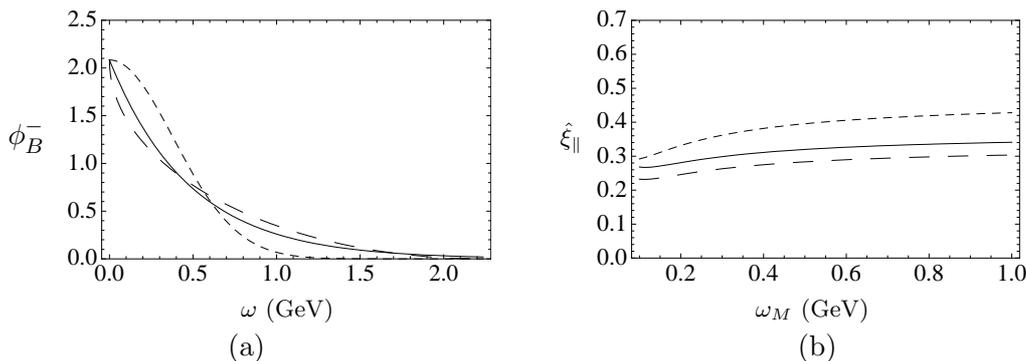
Figure 9: (a) Three models for $\phi_B^-(\omega)$. Solid line: default model (4.1), short-dashed line (4.12), long-dashed line (4.13). (b) The resulting prediction for $\hat{\xi}_\parallel(m_B)$ as a function of the Borel parameter ($\omega_s = 0.4$ GeV, $\mu = 1$ GeV).

the sum rule focuses on small values of ω). But the derivative at $\omega = 0$ takes extreme (but physically not excluded) values 0 and ∞, respectively. In figure 9 we compare the three models for ϕ_B^- as well as the resulting soft form factor $\hat{\xi}_\parallel$ at maximal recoil.

We observe that the effects on the soft form factor are quite sizeable, ranging from about -10% to +25% variations. In order to reduce the associated uncertainty, one clearly needs additional qualitative and quantitative information on the LCDA $\phi_B^-(\omega, \mu)$. In any case, we have to conclude that the sum rule for the soft $B \to \pi$ form factor alone cannot be used to put stringent constraints on $\phi_B^-(0)$.[6]

4.6 Predictions for form factor ratios

As discussed already in our first paper [5], the numerical predictions for individual form factors are quite sensitive to the hadronic input parameters related to the B-meson distribution amplitude and the Borel and threshold parameters. Certainly, part of these uncertainties should cancel in ratios of form factors. In the following, we discuss two kind of ratios: First, we consider ratios of soft form factors for $B \to \pi(K)$ and $B \to \rho(K^*)$ transitions. Second, we consider the corrections to the symmetry relations in the large-energy limit [10], making use of the sum-rule prediction for the factorizable corrections.

4.6.1 Ratios of soft form factors

It is convenient to normalize the soft form factors to the corresponding decay constant and consider the ratios

$$(\xi_\pi/f_\pi) : (\hat{\xi}_\rho^\parallel/f_\rho^\parallel) \qquad \text{and} \qquad (\xi_\rho^\perp/f_\rho^\perp) : (\hat{\xi}_\rho^\parallel/f_\rho^\parallel).$$

In table 1 we present the numerical values obtained from our predictions for the individual form factors at $\mu = 1.0$ GeV, obtained in the previous sections. The quoted uncertainties

---

[6]In [26] independent information on the soft $B \to \pi$ form factor was used to determine the $1/\omega$ moment λ_B of the LCDA $\phi_B^+(\omega)$, by means of the approximate WW relation between $\phi_B^-(0)$ and λ_B (see appendix B). In view of the large dependence on the exact shape of $\phi_B^-(\omega, \mu)$, such a procedure appears questionable to us, at least, the related uncertainty seems to be significantly underestimated in [26].

ratio:	$(\xi_\pi/f_\pi) : (\hat{\xi}_\rho^\parallel/f_\rho^\parallel)$	$(\xi_\rho^\perp/f_\rho^\perp) : (\hat{\xi}_\rho^\parallel/f_\rho^\parallel)$
original sum rule	$1.18^{+0.37}_{-0.32}$	$1.02^{+0.28}_{-0.21}$
modified sum rule	$1.05^{+0.06}_{-0.04} \pm (?)_{\text{syst.}}$	$0.87^{+0.06}_{-0.12} \pm (?)_{\text{syst.}}$

Table 1: Ratios of normalized soft form factors for original and modified sum rules (see text). The errors are obtained by varying the sum-rule parameters (independently for every meson) and ω_0, added in quadrature. For the original sum rule the errors are dominated by the sum rule parameters. For the modified sum rule the errors for the form factor ratios are dominated by the systematic error related to the assumed correlation between the sum rules for the soft form factor and for the decay constants, respectively.

are obtained by varying independently the sum rule parameters for the two considered form factors, together with the ω_0 dependence (the dependence on f_B drops out). Using the original sum rules (2.21), (2.22) with (2.40), the so-obtained uncertainty is dominated by the sum-rule parameters and amounts to about 30%.

Using, on the other hand, the modified sum rules (4.5), (4.8), (4.11) (i.e. assuming a naive correlation between sum rule parameters for soft form factors and decay constants), one gets very stable result close to one, i.e. to the simple approximate relations

$$\xi_P/f_P \simeq \hat{\xi}_V^\parallel/f_V^\parallel \simeq \xi_V^\perp/f_V^\perp \qquad (4.14)$$

which hold in this case, with a parametric uncertainty of about 10-15%. As already said, this error does not include a systematic uncertainty which cannot be estimated in a model-independent way.

SU(3) flavour symmetry corrections: the approximate relations (4.14) can directly be generalized to $B \to K$ and $B \to K^*$ form factors, yielding an estimate for the SU(3) ratios

$$\xi_K/\xi_\pi \simeq f_K/f_\pi \qquad \text{and} \qquad \xi_{K^*}^{\parallel,\perp}/\xi_\rho^{\parallel,\perp} \simeq f_{K^*}^{\parallel,\perp}/f_\rho^{\parallel,\perp}. \qquad (4.15)$$

Notice that the latter ratio is of particular importance for the extraction of the CKM parameter $|V_{ts}/V_{td}|$ from the ratio of the $B \to K^*\gamma$ and $B \to \rho\gamma$ branching fractions [28, 29, 24, 30]. The central values for the SU(3) ratios,

$$\xi_{K^*}^\perp/\xi_\rho^\perp \approx 1.1 - 1.2$$

etc., are thus in line with the naive expectations. However, it is to be stressed that the theoretical uncertainty, to be assigned to the form-factor ratios, depends very strongly on the assumptions about the sum-rule parameters. Of course, we expect a *certain* correlation between the values for s_0 and ω_M in different sum rules. The way how to quantify these correlations (and thus decreasing the uncertainties) appears to us more controversial.
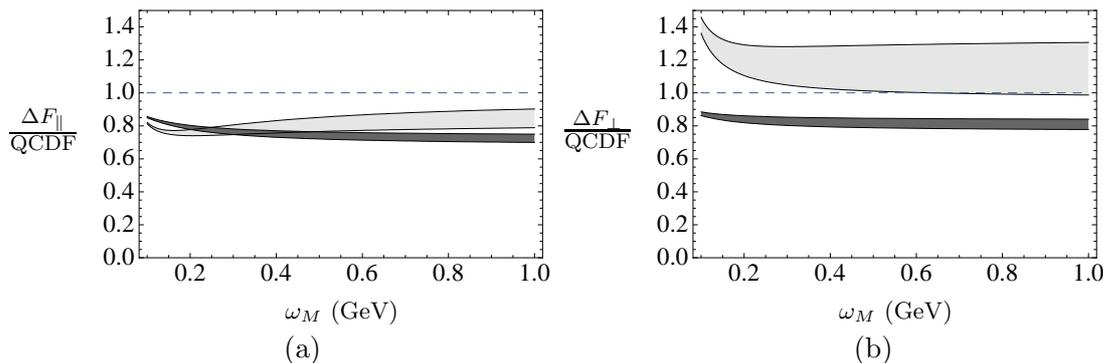
Figure 10: (a) $\Delta F_\parallel$ from (3.22 — light gray band) or (3.24 — dark gray band) normalized to the QCDF result (3.25) as a function of ω_M for $\omega_s \in [0.35, 0.45]\,\text{GeV}$ and $\mu = 1.5\,\text{GeV}$. (b) The same for $\Delta F_\perp$ with $\omega_s \in [0.2, 0.3]\,\text{GeV}$.
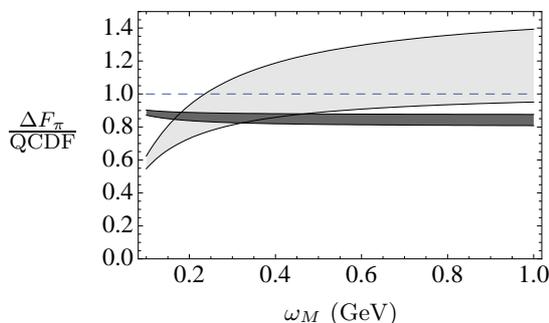
Figure 11: ΔF_π from (3.22 — light gray band) or (3.24 — dark grey band) normalized to the QCDF result (3.25) as a function of ω_M for $\omega_s \in [0.15, 0.25]\,\text{GeV}$ and $\mu = 1.5\,\text{GeV}$.

4.6.2 Corrections to symmetry relations

In figures 10 and 11 we present a quantitative study of the factorizable form-factor contributions, parametrized by the quantities

$$\Delta F_\parallel, \qquad \Delta F_\perp, \qquad \Delta F_\pi.$$

We find it convenient to normalize to the QCD-factorization result (3.25) and compare the original sum rule (3.22) and the modified one (3.24). The threshold parameters ω_s for the three cases are varied within the same ranges we used for the soft form factor analysis, the factorization scale is chosen as $\mu = 1.5\,\text{GeV}$, and the variation with the Borel parameter ω_M is plotted. The following observations can be made:

- The modified sum rule shows a rather mild ω_M dependence, and is typically 15-25% smaller than the QCD factorization result (with asymptotic light-meson DA).

- The original sum rule shows a larger ω_M dependence, and typically lies closer to or even above the QCD factorization result.

- In the case of transverse vector mesons, we observe a rather large deviation between the original sum rule (3.22) and the modified one (3.24). This discrepancy has also been seen in the soft form factor case.

In any case, the sum rule and the QCDF result are in qualitative agreement. In particular, in the sum-rule approach we find no potential source of anomalous enhancement of the ΔF_i as it is sometimes required in phenomenological fits (see, for instance, [31] and references therein). For a more precise quantitative statement, we would also have to consider α_s^2 contributions to the correlation function Π_1, as well as include the α_s and $1/M^2$ corrections in the sum rules for the decay constants, which is beyond the scope of this work.

4.7 Comparison with other work

A study of $B \to \pi(K)$ and $B \to \rho(K^*)$ form factors in the framework of light-cone sum rules involving the B-meson distribution amplitudes also appeared in [32]. The authors use the same set-up with the B-meson treated as an on-shell state in the heavy-mass limit of QCD (which at tree-level coincides with HQET), and the light mesons interpolated by appropriate currents. The correlation functions are calculated only at tree level, but contributions from 3-particle Fock states in the B-meson (which have been neglected in our analysis) are taken into account. The tree-level result for 2-particle contributions to the sum rule coincides with ours for decays into light pseudoscalars and longitudinally polarized vector mesons. In case of transversely polarized vector mesons we used an interpolating tensor current, while the authors of [32] used a vector current, and naturally the results for the corresponding sum rules look different. As discussed above, a disadvantage of the tensor current is the "pollution" by the contribution of a near-lying axial-vector resonance, which is avoided by taking the vector current in [32]. On the other hand, the correlation function with the tensor current is more "natural" from the SCET point of view, since the vector-current correlator turns out to be Λ/m_b-suppressed in the transverse case. Therefore — in contrast to [32] — our sum rule for transversely polarized vector mesons does not receive leading contributions from the 3-particle Fock states at tree-level and takes the same form as in the longitudinal or pseudoscalar case. As a consequence, the leading dependence on the B-meson distribution amplitude drops out in the form-factor ratios considered in table 1.

Certainly, it will be desirable to combine the radiative corrections with the 3-particle contributions to get a more reliable error estimate for the physical $B \to M$ form factors. This includes both, the elimination of the scale ambiguity from $\phi_B^-(\omega, \mu)$ in the tree-level result through α_s corrections to SCET correlation functions, and the effect of Λ/m_b corrections from sub-leading currents and interactions in the SCET Lagrangian. These issues will be left for future work.

5. Summary

In this paper we have studied light-cone sum rules for heavy-to-light form factors within the framework of soft-collinear effective theory (SCET), generalizing our results for the $B \to \pi$ form factors in [5] to the cases of decays into light vector mesons.

The conceptual advantage of our formalism — compared to the more traditional light-cone sum rules in QCD (see e.g. [22, 33 – 35]) — is a clear separation of factorizable ("soft") and non-factorizable ("hard") contributions, on the basis of a systematic expansion in inverse powers of the b-quark mass, already on the level of the correlation functions. This is particularly useful to make contact to the QCD factorization approach, both on the qualitative and on the quantitative level. In the SCET correlation functions, involving interpolating currents for the light mesons under consideration, the hard-collinear dynamics is factorized from the soft dynamics. The latter is described by non-perturbative light-cone wave functions for the B meson. This factorization has been explicitly verified to order α_s accuracy (neglecting 3-particle LCDAs), making use of a new result for the anomalous dimension kernel $\gamma_-(\omega, \omega')$ obtained in [14]. The hard dynamics from scales of order m_b is already integrated out, and appears in matching coefficients of the decay currents in SCET.

Sum rules for the heavy-to-light form factors are, as usual, obtained from a dispersive analysis of the correlation function, introducing non-perturbative parameters associated to the continuum threshold and the Borel transformation. Besides the light-cone distribution amplitude of the B-meson itself, these parameters represent the main source of theoretical (systematic) uncertainties in our numerical predictions for the form factors. In this context, it is important to realize the different dynamical nature of factorizable and non-factorizable form factor contributions, which also reflects itself in the respective sum rules. Considering the sum rule for the factorizable form factor (3.22) in the heavy-quark mass limit, the dependence on the sum-rule parameters factorizes from the soft convolution integral involving the B-meson distribution amplitude. In this case, the sum-rule parameters can be viewed as universal properties of the spectrum of the interpolating current, and by comparing with the sum rule for the meson decay constants, one formally recovers the QCD factorization result.

In contrast, the analogous limit for the non-factorizable form factor contributions (2.40) does not exist beyond tree level, and the responsible non-factorizable logarithmic dependence on the threshold parameter has the same physical origin as the endpoint divergences observed in the standard QCD factorization approach. As a consequence, the sum rule parameters in (2.40) are to be considered as independent non-perturbative input, specific to the soft (i.e. endpoint-dominated) dynamics. (In terms of the interpolating currents in SCET, see (2.9) and (2.10), the factorizable terms probe the first terms with two hard-collinear fields, whereas the non-factorizable terms probe the terms with one hard-collinear and one soft field.) Following a conservative treatment of theoretical systematics, our quantitative estimates for the soft form factor contributions thus leads to rather large uncertainties, where the accepted ranges for the sum rule parameters are constrained by requiring sufficient stability under variations of the Borel parameter, and a sufficiently small continuum pollution. The resulting predictions for the soft form factors are in general agreement with the expectations from QCD light-cone sum rules and QCD factorization. The NLO corrections, calculated in this paper, are numerically significant and can reduce the tree-level estimates by up to 30%. Concerning the energy-dependence of the soft form factors, we find a significant deviation of the simple $1/E^2$ behaviour, in particular for longitudinally

polarized vector mesons.

The uncertainties related to the sum-rule parameters and the B-meson distribution amplitudes can be somewhat reduced by considering form factor *ratios*. Still, a proper error estimate requires to make certain assumptions about correlations between the threshold and Borel parameters in different sum rules. In the simplest case, we find that to first approximation the soft form factors scale with the respective light meson's decay constant.

Acknowledgments

A. Calculation of the imaginary part at one-loop

The imaginary part from diagrams (a1-a4), after $\overline{\text{MS}}$ renormalization, determines the one-loop contribution to the jet function in inclusive $b \to u$ decays [11, 12],

$$
\frac{1}{\pi} \text{Im}[\Pi^a_{\parallel,\perp}] = \frac{\alpha_s C_F}{4\pi} f_B m_B \int_0^\infty d\omega\, \phi_B^-(\omega) \tag{A.1}
$$
$$
\times \left\{ (7 - \pi^2)\, \delta(u) - 3 \left(\frac{1}{u}\right)_*^{[\mu^2/n_+p']} + 4 \left(\frac{\ln[u(n_+p')/\mu^2]}{u}\right)_*^{[\mu^2/n_+p']} \right\},
$$

where $u = (n_- p - \omega + i\eta)$, and the modified plus-distributions are defined through (see e.g. [36]),

$$
\int_{\leq 0}^M du\, F(u) \left(\frac{1}{u}\right)_*^{[m]} = \int_0^M du \frac{F(u) - F(0)}{u} + F(0) \ln\left(\frac{M}{m}\right), \tag{A.2}
$$
$$
\int_{\leq 0}^M du\, F(u) \left(\frac{\ln(u/m)}{u}\right)_*^{[m]} = \int_0^M du \frac{F(u) - F(0)}{u} \ln\frac{u}{m} + \frac{F(0)}{2} \ln^2\left(\frac{M}{m}\right). \tag{A.3}
$$

Using (2.13), taking into account the continuum subtraction $\omega' < \omega_s$, and performing the ω integration in (A.1), this contributes to the sum rule as

$$
\hat{B}[\Pi^a_{\parallel,\perp}](\omega_M) - \text{cont.} = \frac{f_B m_B}{\omega_M} \frac{\alpha_s C_F}{4\pi} \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M} \tag{A.4}
$$

$$\times \left\{ \left(7 - \pi^2 + 3 \ln \left[\frac{\mu^2}{\omega'(n_+ p')} \right] + 2 \ln^2 \left[\frac{\mu^2}{\omega'(n_+ p')} \right] \right) \phi_B^-(\omega') \right.$$

$$\left. + \int_0^{\omega'} d\omega \left(4 \ln \left[\frac{\mu^2}{(\omega' - \omega)(n_+ p')} \right] + 3 \right) \frac{\phi_B^-(\omega') - \phi_B^-(\omega)}{\omega' - \omega} \right\}$$

For convenience, we summarize the results for the imaginary part from the basic structures appearing in the one-loop correlation function:

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega - \omega'} L(\mu) \phi_B^-(\omega) = \int_0^{\omega'} d\omega \frac{\phi_B^-(\omega) - \phi_B^-(\omega')}{\omega - \omega'} + L_0 \phi_B^-(\omega'), \tag{A.5}$$

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega - \omega'} [L(\mu)]^2 \phi_B^-(\omega) = \int_0^{\omega'} d\omega \, (2L_0 - 2L_1) \frac{\phi_B^-(\omega) - \phi_B^-(\omega')}{\omega - \omega'}$$

$$+ \left(L_0^2 - \frac{\pi^2}{3} \right) \phi_B^-(\omega'), \tag{A.6}$$

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega} L_1 \phi_B^-(\omega) = - \int_{\omega'}^\infty d\omega \ln \frac{\omega}{\omega'} \left(\frac{d}{d\omega} \phi_B^-(\omega) \right), \tag{A.7}$$

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega - \omega'} L_1 \phi_B^-(\omega) = - \int_{\omega'}^\infty d\omega \ln \left[\frac{\omega}{\omega'} - 1 \right] \left(\frac{d}{d\omega} \phi_B^-(\omega) \right), \tag{A.8}$$

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega - \omega'} [L_1]^2 \phi_B^-(\omega) = - \int_{\omega'}^\infty d\omega \ln^2 \left[\frac{\omega}{\omega'} - 1 \right] \left(\frac{d}{d\omega} \phi_B^-(\omega) \right) - \frac{\pi^2}{3} \phi_B^-(\omega'), \tag{A.9}$$

and

$$\frac{1}{\pi} \mathrm{Im} \int_0^\infty \frac{d\omega}{\omega - \omega'} L_1 L(\mu) \phi_B^-(\omega) = \int_0^{\omega'} d\omega \, L_1 \frac{\phi_B^-(\omega) - \phi_B^-(\omega')}{\omega - \omega'} \tag{A.10}$$

$$- \int_{\omega'}^\infty d\omega \left(L_0 \ln \left[\frac{\omega}{\omega'} - 1 \right] - \frac{1}{2} \ln^2 \left[\frac{\omega}{\omega'} - 1 \right] - \frac{\pi^2}{6} \right) \left(\frac{d}{d\omega} \phi_B^-(\omega) \right).$$

where

$$L(\mu) = \ln \left[-\frac{\mu^2}{(\omega' - \omega) n_+ p} \right], \qquad L_1 = \ln \left[1 - \frac{\omega}{\omega'} \right], \qquad L_0 = \ln \left[\frac{\mu^2}{\omega' n_+ p} \right],$$

and $\mathrm{Im}[w] < 0$. Notice that the imaginary part resulting from the diagrams (b1+b2) also has support for $\omega > \omega'$.

B. *B*-meson LCDAs in $D \neq 4$ dimensions

We follow the derivation as given in the appendix of [1]. Starting point is the general decomposition of the two-particle B-meson matrix element [9]

$$\langle 0|\bar{q}_\beta(z)\, P(z,0)\, b_\alpha(0)|\bar{B}(p)\rangle = -\frac{if_B M}{4}\left[\frac{1+\slashed{v}}{2}\left\{2\tilde{\phi}_B^+(t,z^2)+\frac{\tilde{\phi}_B^-(t,z^2)-\tilde{\phi}_B^+(t,z^2)}{t}\slashed{z}\right\}\gamma_5\right]_{\alpha\beta},$$
(B.1)

where $v\cdot z = t$ and, for the moment, we allowed the separation between the quark fields to be off the light-cone, $z^2 \neq 0$. $P(z,0)$ is the usual path-ordered exponential which, however, will be neglected in the derivation of the Wandzura-Wilczek (WW) relation.

The equation of motion for the light spectator quark relates $\tilde{\phi}_B^-(l_+)$ to $\tilde{\phi}_B^+(l_+)$ in the approximation that the three-particle amplitudes are set to zero (the effect of three-particle LCDAs in $D \neq 4$ is discussed in [14]). We require the right-hand side of eq. (B.1) to vanish after application of $[\slashed{\partial}_{z^2}]_{\beta\gamma}$ (which is true only if the three-particle Fock-state $b\bar{q}g$ is neglected), and $\tilde{\phi}_\pm^B(t,z^2)$ not to vanish as $z^2 \to 0$. Writing

$$\partial_z^\mu = v^\mu\,\partial_t + 2z^\mu\,\partial_{z^2}\,,$$

we obtain two constraints from the two independent Dirac structures $\frac{1+\slashed{v}}{2}\gamma_5$ and $\frac{1+\slashed{v}}{2}\gamma_5\slashed{z}$

$$\partial_t\tilde{\phi}_B^- + \frac{D-2}{2t}\,(\tilde{\phi}_B^- - \tilde{\phi}_B^+)\Big|_{z^2=0} = 0$$
(B.2)

$$-\partial_t(\tilde{\phi}_B^- - \tilde{\phi}_B^+) + 4t\,\partial_{z^2}\tilde{\phi}_B^+ + \frac{1}{t}\,(\tilde{\phi}_B^- - \tilde{\phi}_B^+)\Big|_{z^2=0} = 0\,.$$
(B.3)

The first relation modifies the WW relation as quoted in (110) of [1]. Inserting the t-derivative of the first equation into the second one, the latter can be simplified as

$$\frac{\partial\tilde{\phi}_B^+}{\partial z^2} + \frac{1}{2(D-2)}\frac{\partial^2\tilde{\phi}_B^-}{\partial t^2}\Big|_{z^2=0} = 0\,,$$
(B.4)

which modifies equation (111) in [1]. The momentum-space representation of (B.2) now reads

$$\int_0^{l_+} d\eta\,(\phi_B^-(\eta) - \phi_B^+(\eta)) = \frac{2}{D-2}\,l_+\phi_B^-(l_+) \qquad \text{or} \qquad \phi_B^+(l_+) = -\frac{2}{D-2}\,l_+\,\phi_B^{\prime-}(l_+),$$
(B.5)

which is solved by

$$\phi_B^-(l_+) = \frac{D-2}{2}\int_0^1\frac{d\eta}{\eta}\,\phi_B^+(l_+/\eta).$$
(B.6)

The factor $2/(D-2)$ also appears in the momentum-space projector for the B-meson wave function in the WW approximation

$$M_{\beta\alpha}^B = -\frac{if_B M}{4}\left[\frac{1+\slashed{v}}{2}\left\{\phi_B^+(\omega)\,\slashed{n}_+ + \phi_B^-(\omega)\,\slashed{n}_- - \frac{2}{D-2}\,\omega\phi_B^-(\omega)\,\gamma_\perp^\mu\frac{\partial}{\partial l_{\perp\mu}}\right\}\gamma_5\right]_{\alpha\beta}.$$
(B.7)

where the transverse Dirac matrices $\gamma_\perp$ obey $\gamma_\perp^\mu\gamma_\mu^\perp = D - 2$.

References

[1] M. Beneke and T. Feldmann, *Symmetry-breaking corrections to heavy-to-light B meson form factors at large recoil*, *Nucl. Phys.* **B 592** (2001) 3 [hep-ph/0008255].

[2] M. Beneke and T. Feldmann, *Factorization of heavy-to-light form factors in soft- collinear effective theory*, *Nucl. Phys.* **B 685** (2004) 249 [hep-ph/0311335].

[3] R.J. Hill and M. Neubert, *Spectator interactions in soft-collinear effective theory*, *Nucl. Phys.* **B 657** (2003) 229 [hep-ph/0211018];
B.O. Lange and M. Neubert, *Factorization and the soft overlap contribution to heavy-to-light form factors*, *Nucl. Phys.* **B 690** (2004) 249 [hep-ph/0311345].

[4] C.W. Bauer, D. Pirjol and I.W. Stewart, *Factorization and endpoint singularities in heavy-to-light decays*, *Phys. Rev.* **D 67** (2003) 071502 [hep-ph/0211069].

[5] F. De Fazio, T. Feldmann and T. Hurth, *Light-cone sum rules in soft-collinear effective theory*, *Nucl. Phys.* **B 733** (2006) 1 [hep-ph/0504088].

[6] T. Hurth, F. De Fazio and T. Feldmann, *The $B \to \pi$ form factor from light-cone sum rules in soft-collinear effective theory*, PoS(HEP2005)215 [hep-ph/0509167].

[7] C.W. Bauer, S. Fleming, D. Pirjol and I.W. Stewart, *An effective field theory for collinear and soft gluons: heavy to light decays*, *Phys. Rev.* **D 63** (2001) 114020 [hep-ph/0011336];
C.W. Bauer and I.W. Stewart, *Invariant operators in collinear effective theory*, *Phys. Lett.* **B 516** (2001) 134 [hep-ph/0107001].

[8] M. Beneke, A.P. Chapovsky, M. Diehl and T. Feldmann, *Soft-collinear effective theory and heavy-to-light currents beyond leading power*, *Nucl. Phys.* **B 643** (2002) 431 [hep-ph/0206152];
M. Beneke and T. Feldmann, *Multipole-expanded soft-collinear effective theory with non-abelian gauge symmetry*, *Phys. Lett.* **B 553** (2003) 267 [hep-ph/0211358].

[9] A.G. Grozin and M. Neubert, *Asymptotics of heavy-meson form factors*, *Phys. Rev.* **D 55** (1997) 272 [hep-ph/9607366].

[10] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynal, *Heavy-to-light form factors in the heavy mass to large energy limit of QCD*, *Phys. Rev.* **D 60** (1999) 014001 [hep-ph/9812358].

[11] C.W. Bauer and A.V. Manohar, *Shape function effects in $B \to X_s\gamma$ and $B \to X_u l\nu$ decays*, *Phys. Rev.* **D 70** (2004) 034024 [hep-ph/0312109].

[12] S.W. Bosch, B.O. Lange, M. Neubert and G. Paz, *Factorization and shape-function effects in inclusive B-meson decays*, *Nucl. Phys.* **B 699** (2004) 335 [hep-ph/0402094].

[13] B.O. Lange and M. Neubert, *Renormalization-group evolution of the B-meson light-cone distribution amplitude*, *Phys. Rev. Lett.* **91** (2003) 102001 [hep-ph/0303082].

[14] T. Feldmann and G. Bell, *Light-cone distribution amplitudes for non-relativistic bound states*, arXiv:0711.4014; *Modelling light-cone distribution amplitudes from non-relativistic bound states*, *Siegen preprint* SI-HEP-2007-20, to be published.

[15] G. Bell and T. Feldmann, *Heavy-to-light form factors for non-relativistic bound states*, *Nucl. Phys.* **164** *(Proc. Suppl.)* (2007) 189 [hep-ph/0509347].

[16] M. Beneke and D. Yang, *Heavy-to-light B meson form factors at large recoil energy: spectator scattering corrections*, *Nucl. Phys.* **B 736** (2006) 34 [hep-ph/0508250].

[17] G.G. Kirilin, *Loop corrections to the form factors in $B \to \pi l \nu$ decay*, hep-ph/0508235.

[18] T. Becher and R.J. Hill, *Loop corrections to heavy-to-light form factors and evanescent operators in SCET*, *JHEP* **10** (2004) 055 [hep-ph/0408344].

[19] S.J. Lee and M. Neubert, *Model-independent properties of the B-meson distribution amplitude*, *Phys. Rev.* **D 72** (2005) 094028 [hep-ph/0509350].

[20] V.M. Braun, D.Y. Ivanov and G.P. Korchemsky, *The B-meson distribution amplitude in QCD*, *Phys. Rev.* **D 69** (2004) 034014 [hep-ph/0309330].

[21] P. Ball and V.M. Braun, *The ρ meson light-cone distribution amplitudes of leading twist revisited*, *Phys. Rev.* **D 54** (1996) 2182 [hep-ph/9602323].

[22] P. Ball and V.M. Braun, *Exclusive semileptonic and rare B meson decays in QCD*, *Phys. Rev.* **D 58** (1998) 094016 [hep-ph/9805422].

[23] P. Ball and R. Zwicky, $B_{d,s} \to \rho, \omega, K^*, \phi$ *decay form factors from light-cone sum rules revisited*, *Phys. Rev.* **D 71** (2005) 014029 [hep-ph/0412079].

[24] M. Beneke, T. Feldmann and D. Seidel, *Exclusive radiative and electroweak $b \to d$ and $b \to s$ penguin decays at NLO*, *Eur. Phys. J.* **C 41** (2005) 173 [hep-ph/0412400].

[25] R.J. Hill, *Heavy-to-light meson form factors at large recoil*, *Phys. Rev.* **D 73** (2006) 014012 [hep-ph/0505129].

[26] A. Khodjamirian, T. Mannel and N. Offen, *B-meson distribution amplitude from the $B \to \pi$ form factor*, *Phys. Lett.* **B 620** (2005) 52 [hep-ph/0504091].

[27] J. Chay, C. Kim and A.K. Leibovich, *SCET sum rules for heavy-to-light form factors*, *Phys. Lett.* **B 628** (2005) 57 [hep-ph/0508157].

[28] S.W. Bosch and G. Buchalla, *The radiative decays $B \to V\gamma$ at next-to-leading order in QCD*, *Nucl. Phys.* **B 621** (2002) 459 [hep-ph/0106081].

[29] A. Ali and A.Y. Parkhomenko, *Branching ratios for $B \to \rho\gamma$ decays in next-to- leading order in α_s including hard spectator corrections*, *Eur. Phys. J.* **C 23** (2002) 89 [hep-ph/0105302].

[30] P. Ball and R. Zwicky, $B \to K^*\gamma$ *vs* $B \to \rho\gamma$ *and* $|V_{td}/V_{ts}|$, hep-ph/0608009.

[31] C.W. Bauer, *Hadronic B decays from SCET*, hep-ph/0606018.

[32] A. Khodjamirian, T. Mannel and N. Offen, *Form factors from light-cone sum rules with B-meson distribution amplitudes*, *Phys. Rev.* **D 75** (2007) 054013 [hep-ph/0611193].

[33] A. Khodjamirian and R. Ruckl, *QCD sum rules for exclusive decays of heavy mesons*, *Adv. Ser. Direct. High Energy Phys.* **15** (1998) 345 [hep-ph/9801443].

[34] P. Ball, $B \to \pi$ *and* $B \to K$ *transitions from QCD sum rules on the light-cone*, *JHEP* **09** (1998) 005 [hep-ph/9802394].

[35] P. Colangelo and A. Khodjamirian, *QCD sum rules: a modern perspective*, hep-ph/0010175.

[36] F. De Fazio and M. Neubert, $B \to X_u \ell \bar{\nu}_\ell$ *decay distributions to order α_s*, *JHEP* **06** (1999) 017 [hep-ph/9905351].